Article

Estimating Output Gap in Japan: A Latent Variable Approach

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Summary

Since the late 1990s, Japanese economy was in deflation for more than twenty years. Recently, there have been some signs of ending deflation. Deflation is usually defined as sustained decline of prices frequently associating economic stagnation. The economic stagnation can be measured by various means, and this paper among those explores measurement of the output gap or the GDP gap. At first, the study organizes the measurement methods for estimating the output gap such as the production function approach, the univariate approach employing mechanical filters, the empirical approach based on Okun's law, and the structural vector autoregression (SVAR) approach using latent variables. The paper adopts one of the latent variable approaches employing the state space model based on Kuttner (1994) and tries to estimate Japanese output gap from mid-1990s comparable with output gap measured by other methods.

JEL Classifications: C32, C53, and E32

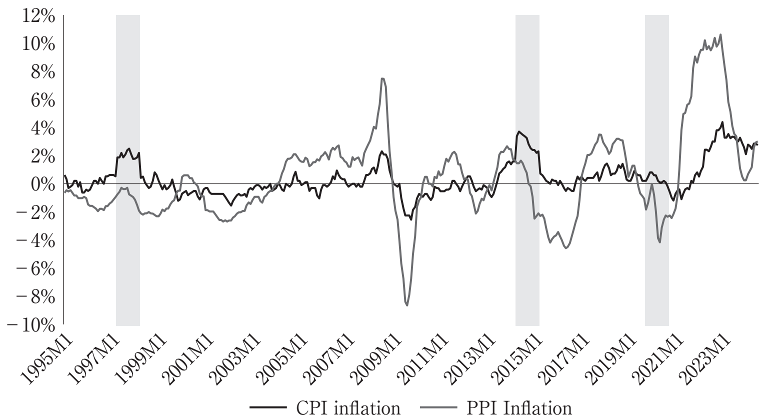
Key Words: Output Gap, State Space Model, Business Cycle, Japan

1. Introduction

In Japan, the bubble economy collapsed in early 1990s, and the deflation started in late 1990s continuously to recent years. In 2024, *Annual Report on the Japanese Economy and Public Finance 2023* (CAO, 2024) analyzes Japan is now ending deflation. Chart 1 depicts (1) the GDP growth and (2) the inflation rate based on the consumer price index (CPI) and the producer price index (PPI).

Chart 1: (1)GDP Growth

(2) Inflation rate measured by CPI and PPI



note: (1)GDP Growth is quarter-on-quarter changes based on seasonally adjusted series while inflation is year-on-year changes.
(2) The PPI inflation eliminates consumption tax while the CPI inflation includes tax effects.
(3) The shadowed area indicates consumption tax effect periods.
source: author based on SNA, CPI and PPI statistics

After the 1997 hike on the consumption tax rate in April, Japanese inflation began to decline to negative continuing until 2008 with some exceptions. A lot of economics literatures points out negative effect of deflation on demand（Ito and Mishkin, 2006, Mckinnon, 2007, Svensson, 2003, Baba et al., 2007[1]). Recently, due to worldwide commodity prices hike, especially those of energy and foodstuff, Japanese prices indicate ending the deflation at the headline CPI inflation rate around three percent. Usually, many economists think that one of the most important purposes of economic management is to maximize growth or minimize unemployment under the price stability. And the Phillips curve shows the relationship between unemployment and prices.

The correlation between prices movements and GDP gap seems strong. For macroeconomic stability, policy makers have to focus strongly on output gap to avoid both inflation and deflation.[2] The study surveys some methods for estimating output gap such as the production function approach, the univariate approach employing mechanical filters, the empirical approach based on Okun's law, and some more advanced econometric approaches using the latent variables. Among those, the study tries to estimate Japanese output gap from mid-1990s employing the state space model based on Kuttner (1994). The paper consists of five chapters including this introduction part: the second chapter organizes some methodologies of measuring output gap; the third chapter focuses on the detailed state space model approach among those; the fourth chapter reveals data and estimation results; and the final chapter briefly concludes the discussion.

2. Methodologies of Measuring Output Gap

GDP i.e., output consists of four elements equally to other economic indicators as follows:

(EQ-1) output and its decomposition

$$Y_t = C_t + T_t + S_t + E_t$$

Where Y actual output
C cyclical factor
T trend factor
S seasonal factor
E error

When we adopt seasonal adjusted or annual data, it is not necessary to consider seasonal movement. Error term is also regarded as white noise to be ignored. Many economists consider that in a short term, the cyclical factor is dominant while the trend factor is more important in a long run. So the trend factor forms potential movement and the cyclical factor implies output gap mainly based on business cycle. Removing negligible factors such as seasonal factor and error term, and adopting different notation, we obtain following equation instead of (EQ-1):

(EQ-2) definition of output gap

$$Y_t = Y^*_t + G_t \quad \text{or} \quad G_t = Y_t - Y^*_t$$

Where Y^* potential output
G output gap

In natural, the output gap is defined as difference between potential and actual outputs. Some methodologies for measuring the output gaps are reported as research results of the central bank and/or government as well as international organizations: e.g., Yoshida (2017) of the Cabinet Office of the Japanese government, Kawamoto et al. (2017) of the Bank of Japan, OBR (2011) of the UK government, Guillochon and Le Roux (2023) of the Banques de France, Bassanetti et al. (2010) of the Banca d'Italia, Kim and Moon (2000) of the Bank of Korea, de Brouwer (1998) of the Reserve Bank of Australia, Claus (1999) of the Reserve Bank of New Zealand, Álvarez and Gómez-Loscos (2018)[3] of the Banco de España, Fedderke and Daniel (2016) of the South African Reserve Bank, Dupasquier (1997) and St-Amant and van Norden (1997) of the Bank of Canada, Fuentes et al. (2007) of the Banco Central de Chile, Sarikaya et al. (2005) of the Central Bank of the Republic of Turkey, Yoshioka (2002)[4] of BAPPENAS of the Indonesian government, Gradzewicz and Kolasa (2004)[5] of the National Bank of Poland, Cerra and Saxena (2000) and Alichi (2015) of the IMF, Chalaux and Guillemette (2019) of the OECD, Havik et al. (2014) of European Commission, Morley et al. (2023)[6] of the European Central Bank, and so on. In addition, it is worth to stress that BIS (1997) features the relationship between output gap and inflation, with various analyses reported by member countries.[7] The central banks and governments, as well as international organizations, pay deep attention on the output gap as one of the most important indicators when managing macroeconomy. Several approaches can be applicable for estimating the output gap such as the production function approach, the univariate approach employing mechanical filters, the empirical approach based on Okun's law, and some more advanced econometric approaches using latent variables in the SVAR.

2.1 Production Function Approach

A lot of economists seem to have broad consensus that the production function approach is one of the best or the most reliable measures to estimate the output gap if the function is appropriately estimated. This approach is based on the following model:

(EQ-3) production function[8]

$$Y_t = f(A_t,\ K_t,\ L_t)$$

$$Y^*_t = f(A_t,\ K^*_t,\ L^*_t)$$

where
- f production function
- K utilized capital stock
- K^* potential or total capital stock
- L actual labor input
- L^* potential labor input

It is not so difficult to introduce some observable data as follows:

(EQ-4) output gap

$$G_t = Y_t - Y^*_t = h(A_t,\ cu_t,\ H_t,\ H^*_t,\ U_t,\ U^*_t)$$

where
- h output gap function
- cu capacity utilization rate of capital stock
- H actual labor hours

H^* potential labor hours[9]

U actual unemployment rate (decimal)

U^* natural unemployment rate (decimal)

or $L^*_t = L_t \cdot \frac{H^*_t}{H_t} \cdot \frac{1-U^*_t}{1-U_t-U^*_t}$

The signs of the partial differential coefficients from the 2nd to the 6th elements in the output gap function of (EQ-4) are assumed as follows while that of the total factor productivity is not decided in unique:

(EQ-5) signs of partial differential coefficients

$$h_2 = \frac{\partial G}{\partial cu} > 0,\ h_3 = \frac{\partial G}{\partial H} > 0,\ h_4 = \frac{\partial G}{\partial H^*} < 0$$

$$h_5 = \frac{\partial G}{\partial U} < 0,\ h_6 = \frac{\partial G}{\partial U^*} > 0$$

Although this approach based on the production function is theoretically one of the most reliable methods and actually adopted in various developed countries, we find two types of difficulties: observation of latent variables and data availability. For this approach, two latent variables are required such as potential labor hours (H^*) and natural rate of unemployment (U^*) other than potential or total capital stock (K^*). The accuracy of estimated result depends highly on the assumptions of these latent variables. Moreover, related to data availability, the capacity utilization rate (cu) and the capital stock are not fully available. In a lot of developed countries, it seems that the capacity utilization data in the manufacturing sector are available. Those in the service sector, however, are not fully available. On the other hand, in some developing countries, the capital stock data are not available, or at best, not so reliable. It could be a very hard task to calibrate these actual and latent data for accurate estimation of the potential output and the output gap.

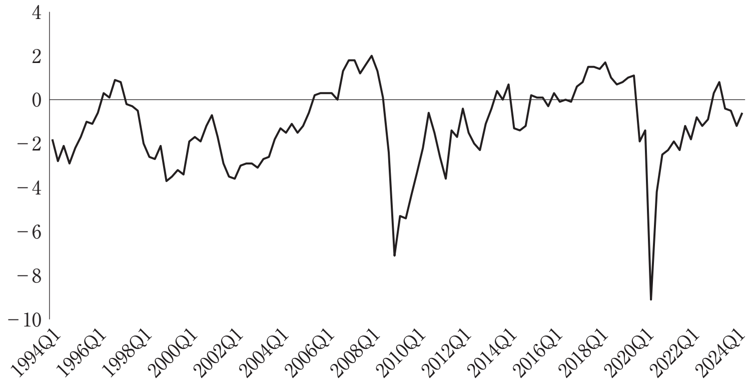
In Japan, both the Cabinet Office (CAO) of the government and the Bank of Japan (BOJ) of central bank reveal the estimation results of the output gap. Both employ the production function approach for the estimation. Chart 2 reports the estimation results as of GDP percent.

2.2 Univariate Approach

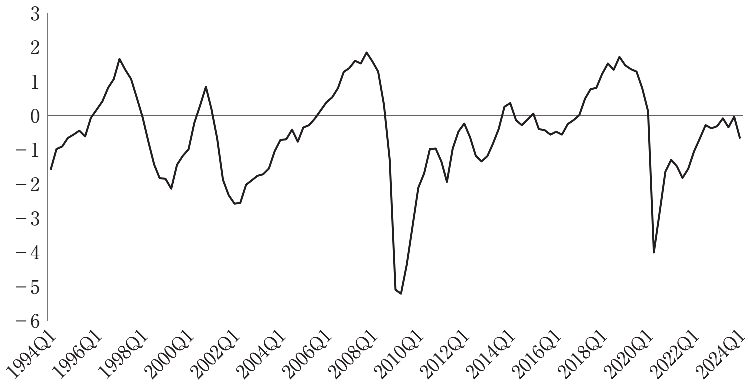
Until 1970s, one of the simplest methods to measure the output gap was to regress time-series variables with a linear time trend. Schumpeter (1935) refers peak-to-peak or trough-to-trough method as counting business cycle. Even at present, it is applicable for understanding the output gap at a glance. Later, Hodrick and Prescott (1981, 1997) introduced a more refined way of dividing cyclical and trend factors from timeseries variables. Their method is widely called Hodrick-Prescott filter, minimizing the following value of HP:

Chart 2: Output Gap

(1) Cabinet Office



(2) Bank of Japan



source: (1) CAO data (https://www5.cao.go.jp/keizai3/getsurei/2421gap.xlsx accessed on September 5, 2024)
(2) BOJ data (https://www.boj.or.jp/research/research_data/gap/gap.xlsx accessed on September 5, 2024)

(EQ-6) Hodrick-Prescott filter

$$HP=\Sigma_{i=1}^{t}(C_i)^2+\lambda\Sigma_{i=2}^{t-1}[(T_{i+1}-T_i)-(T_i-T_{i-1})]^2$$

where λ smoothing parameter

Related to a smoothing parameter, Hodrick and Prescott (1981) recommend the value of 100 for annual data, 1600 for quarterly data, and 14400 for monthly data. In general, it is regarded that the larger this parameter is, the smoother the obtained trend is.

The Hodrick-Prescott filter appears to be a more sophisticated way to divide economic and other timeseries data into cyclical and trend factors than simple regression with a linear time trend. A lot of economists criticize that this kind of mechanical filters do not stand on any theoretical basis of economics. On the other hand, this approach, including other similar methods, is still adopted for many empirical analyses such as Guay and St-Amant (1996, 1997), St-Amant and van Norden (1997), Marcet and Ravn (2003), and so on. Moreover, Taylor (1999) employs the Hodrick-Prescott filter for calculation of the output gap when applying the Taylor rule for historical monetary policy evaluation.[10] Drehmann and Yetman (2018) insist that the Hodrick-Prescott filter is applicable for measuring the credit gap. In Bank of Mexico, Cuadra (2008) reveals the Hodrick-Prescott filter is utilized to derive the trend. It seems that this kind of mechanical filters are applied practically in some central banks and governments.

2.3 Empirical Approach

This approach adopts some empirical relationship between the output gap and other economic data. One of the most diffused and famous methods of this approach is Okun's law insisted in Okun (1962). Based on a stable relationship between GDP and unemployment rate, Okun's law is to be formulated as follows:

(EQ-7) Okun's law

$$Y^*_t = Y_t[1+\theta(U_t - U^*_t)]$$

where θ coefficient parameter (decimal)

Okun (1962) proposes that $\theta = 3.2$ for U.S. economy. For Japanese economy Hamada and Kurosawa (1984) adopt Okun's law to investigate relation between production and unemployment. This approach, however, contains the same difficulty of the latent variable mentioned at the production function part. In the study, the concept of the Okun's law is employed later.

2.4 Structural Vector Autoregression (SVAR) Approach

One of the most significant features of the SVAR process can be found in a unit root process. This implies that the actual output can be decomposed into a permanent component and a transitory one. The SVAR methodology with long-run restrictions proposed by Blanchard and Quah (1989) is usually adopted to obtain an estimate of these components. According to Claus (1999), the Reserve Bank of New Zealand employed this methodology to estimate the output gap as an important indicator for its monetary policy management until mid-2010s.[11] Claus (1999) addresses that the output, employment and capacity utilization are written as a linear combination of current and past structural shocks as follows:

(EQ-8) structural VAR process

$$\begin{bmatrix} \Delta Y_t \\ l_t \\ cu_t \end{bmatrix} = \begin{bmatrix} S_{11}(L) & S_{12}(L) & S_{13}(L) \\ S_{21}(L) & S_{22}(L) & S_{23}(L) \\ S_{31}(L) & S_{32}(L) & S_{33}(L) \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \\ \nu_{3t} \end{bmatrix}$$

where l employment

$S_{ij}(L)$ polynomials in the lag operator

ν_{it} uncorrected white noise disturbances or structural shocks

The shocks here are normalized such that var $(\upsilon_{it})=1$ $(i=1, 2, 3)$ but not observed. According to Claus (1999), we thus have to execute the estimation in unrestricted form of VAR to identify the structural mode at first and in restricted form at second. Here, restrictions are given that the demand-shocks have temporary effects on the output while its cumulative effects in long-run must be identical to zero. According to methodology of Blanchard and Quah (1989) under the assumption that the innovations in both restricted and unrestricted forms are a linear combination of the structural shocks, we finally obtain both potential output and output gap as follows:[12)]

(EQ-9) potential output and the output gap

$$\Delta Y^*_t = S_{11}(L)\upsilon_{it} \quad \text{and}$$

$$G_t = S_{12}(L)\upsilon_{2t} + S_{13}(L)\upsilon_{3t}$$

One of the most advantageous points of this methodology is that neither the potential output nor the output gap is necessarily restricted to follow a random walk, an autoregressive, or other specific stochastic processes. However, we possibly face a difficulty of data availability of employment and capacity utilization in some countries.

Because of these difficulty and criticism, the paper tries to employ a state space approach to measure the output gap in Japan for application to developing countries

3. State Space Model Approach

The paper mainly adopts the methodology proposed in Kuttner (1984) to measure the output gap based on a state space model.[13)] First, a stochastic model is assumed as follows:

(EQ-10) theoretical stochastic model

$$y^*_t = \mu_t + y^*_{t-1} + \varepsilon^y_t$$

$$\mu_t = \mu_{t-1} + \varepsilon^\mu_t$$

$$g_t = \Sigma\phi_k g_{t-k} + \varepsilon^g_t$$

$$\pi_t = \alpha + \Sigma\beta_k \pi_{t-k} + \gamma g_t + \varepsilon^\pi_t$$

where

y^*	log potential output (GDP) i.e., $y^*_t = \log(Y^*_t)$
μ	drift for log potential output
g	log output gap
π	inflation rate
ε^i	error of each item ($i = y^*, \mu, g, \pi$)
$\alpha, \beta_k, \varphi_k, \gamma$	parameters ($k=1, 2, 3, \cdots$)

The first and the second equations of (EQ-10) imply that the potential output is assumed to follow a random walk process with a drift, which also follows a random walk process. The third of (EQ-10) does that the output gap follows an autoregressive process of a certain order. The formulation of this equation is mainly based on Watson (1986) and Kuttner (1994)[14]. The fourth of (EQ-10) is based on Phillips curve mentioned later. Since this model consists of an observed equation and three transit equations, we have to transform this to another practical model that consists of two observed equations and two transit equations later.

After overviewing a state space model, the Phillips curve is focused on. The original Phillips curve empirically describes a negative correlation between money wage growth and unemployment. It is now being applied to the relationship between inflation and unemployment. When the markup pricing mechanism is dominant, the Phillips curve is expressed as follows:

(EQ-11) Phillips curve

$$\frac{\Delta w_t}{w_t} \approx \frac{\Delta p_t}{p_t} = \pi_t = -\lambda_i U_t$$

where w money wage

p prices

λ_1 parameter

Introducing natural unemployment rate and inflation expectation to the model, we obtain following equation:

(EQ-12) inflation

$$\pi_t = \pi^*_t - \lambda_0 (U_t - U^*_t)$$

The equation of the Okun's law referred at (EQ-7) is to be transformed, taking logarithm for the output, as follows:

(EQ-13) Okun's law transformed

$$y^*_t - y_t = \theta (U_t - U^*_t)$$

where y_t log actual output (GDP) i.e., $y_t = \log(Y_t)$

Substituting (EQ-13) into (EQ-12), we obtain following equation of inflation:

(EQ-14) inflation transformed

$$\pi_t = \pi^*_t - \gamma (y^*_t - y_t) = \pi^*_t - \gamma g_t$$

where $\gamma = -\dfrac{\lambda_1}{\theta}$

Since it is plausible to assume that the expected inflation rate is subject partly to static expectation, we thus obtain the following equation:

(EQ-15) static expectation of inflation

$\pi^*_t = \tau_0 + \Sigma\tau_k\pi_{t-k}$

where τ parameters ($i=1, 2, 3, \cdots$) or intercept ($i=0$)

The equation of (EQ-15) can be substituted into (EQ-14), we thus obtain a new inflation equations as follows:

(EQ-16) inflation, new

$\pi_t = \alpha + \Sigma\tau_k\pi_{t-k} + \gamma g_t + \varepsilon^\pi_t$

where $\alpha = \lambda_0\tau_0$

$\beta_k = \lambda_0\tau_k$ ($k=1, 2, 3, \cdots$)

Checking that (EQ-16) is identical to the third equation of (EQ-10) in the stochastic model, we have completed the development of theoretical model. Second, the practical estimation methodology is to be considered. The state space model consists of both observation equation and transit equation. The latter involves unobservable state variables. After confirming the theoretical base of the stochastic model for the output gap, we then consider practically computable state space model because the theoretical state space model of (EQ-10) contains an observed equation for inflation rate, and three transit equations for potential output, output gap, and drift for potential output. It is thus required to transform one of the transit equations to an observed one. For this purpose, the difference of the log actual or observable output is to be introduced as follows:

(EQ-17) difference of log actual output[15]

$\Delta y_t = y_t - y_{t-1} = [y^*_t + g_t] - [y^*_{t-1} + g_{t-1}]$

The first equation of (EQ-10) is to be substituted into the right hand of (EQ-17). We obtain following equation:

(EQ-18) difference of log actual output, transformed[16]

$\Delta y_t = [(\mu_t + y^*_{t-1} + \varepsilon^y_t) + g_t] - [y^*_{t-1} + g_{t-1}]$ then

$\Delta y_t = \mu_t + (g_t - g_{t-1}) + \varepsilon^y_t$

Now it is apparent that we can use the second equation of (EQ-18) instead of the first equation of (EQ-10). Ignoring the duplication, the newly transformed state space model in mathematical terms is as follows:

(EQ-19) state space model, transformed

$\Delta y_t = \mu_t + (g_t - g_{t-1}) + \varepsilon^y_t$

$\pi_t = \alpha + \Sigma\beta_k\pi_{t-k} + \gamma g_t + \varepsilon^\pi_t$

$$g_t = \Sigma \phi_k g_{t-k} + \varepsilon^g{}_t$$

$$\mu_t = \mu_{t-1} + \varepsilon^\mu{}_t$$

According to the existing studies of Watson (1986) and Kuttner (1994), it is plausible to assume that the output gap follows the autoregression process of order two, and the inflation expectation follows the previous period.[17] The state space model is expressed as follows:

(EQ-20) observation equations

$$\begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ \gamma & 0 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ g_{t-1} \\ \mu_t \end{bmatrix} + \begin{bmatrix} \varepsilon^y{}_t \\ \varepsilon^\pi{}_t \end{bmatrix}$$

(EQ-21) transit equations

$$\begin{bmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ \mu_t \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ g_{t-2} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^g{}_t \\ 0 \\ 0 \\ \varepsilon^\mu{}_t \end{bmatrix}$$

Both theoretical overview of the state space model and practical transformation of the model are completed. It is the time to estimate the output gap to employ the model.

4. Data and Estimation Results

The study employs following procedure for estimation of the output gap in Japan:

(1) The seasonal adjusted series of GDP at constant prices of 2020 published by the Cabinet Office is utilized as a proxy of the output.[18]

(2) The inflation rate is measured based on quarter-to-quarter change using the Producer Price Index (PPI) excluding consumption tax of the Corporate Goods Price Index published by the Bank of Japan.[19] The seasonal adjustment is completed with default options of EViews.

(3) Data are employed available on September 5, 2024.

(4) EViews V12 carries out estimation.

Table 1 and chart 3 report descriptive statistics of GDP and PPI and results of unit roots test for their first order differences.

Table 2 reports estimation results of the state space model consisting of observation equations and transit equations while Chart 4 depicts the estimated output gap as a percent of GDP.

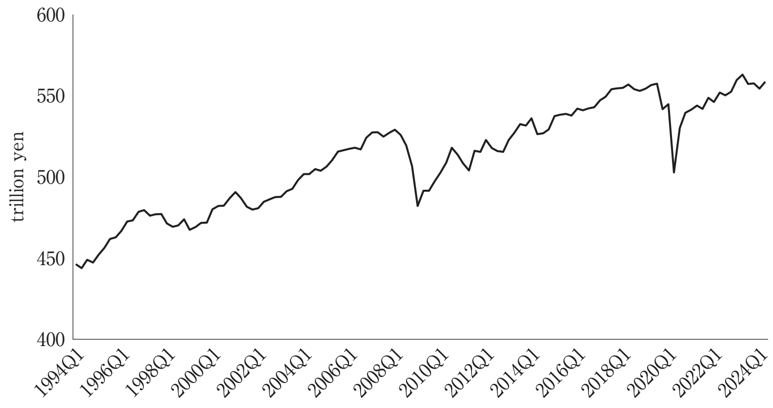
Table 1: Descriptive Statistics

	GDP	PPI
Mean	512337.7	103.1
Median	516135.3	102.0679
Maximum	562988.1	122.1962
Minimum	443815	95.58265
Std. Dev.	31477.83	5.522257
Skewness	−0.220581	1.69886
Kurtosis	1.930186	6.125984
Unit Root Test	−12.49737 (0.00000)	5.66328 (0.00000)

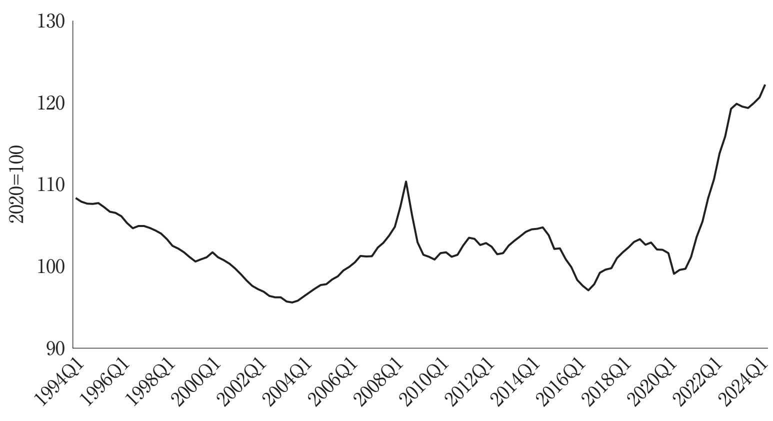
source: author based on SNA statistics of the CAO and PPI data of the BOJ
note: The row of Unit Root Test indicates t-statistics without parentheses and probability with parentheses, executed with log first order series based on Dickey and Fuller (1979, 1981).

Chart 3: GDP and PPI data

(1) GDP



(2) PPI



source: author based on SNA statistics of the CAO and PPI data of the BOJ

Table 2: Estimation Results

(1) Output (GDP)

Variable	Coefficient	Std. Error	t-Statistic
constant	0.491246	1.13E−12	4.35E+11
g	0.101804	6.02E−13	1.69E+11
g_{-1}	−9.00E−07	4.52E−14	−19907706
μ	1	2.31E−12	4.34E+11
R-squared	1	Mean dependent var	0.001917
Adjusted R-squared	1	S.D. dependent var	0.0131
S.E. of regression	3.23E−13	Sum squared resid	1.21E−23
F-statistic	6.51E+22	Durbin-Watson stat	1.393604

(2) PPI Inflation

Variable	Coefficient	Std. Error	t-Statistic
constant	0.001375	5.55E−14	2.48E+10
g	0.180389	1.10E−12	1.64E+11
π_{-1}	0.500655	5.78E−12	8.66E+10
R-squared	1	Mean dependent var	0.001037
Adjusted R-squared	1	S.D. dependent var	0.010349
S.E. of regression	6.07E−13	Sum squared resid	4.31E−23
F-statistic	1.73E+22	Durbin-Watson stat	1.010268

(3) Output Gap

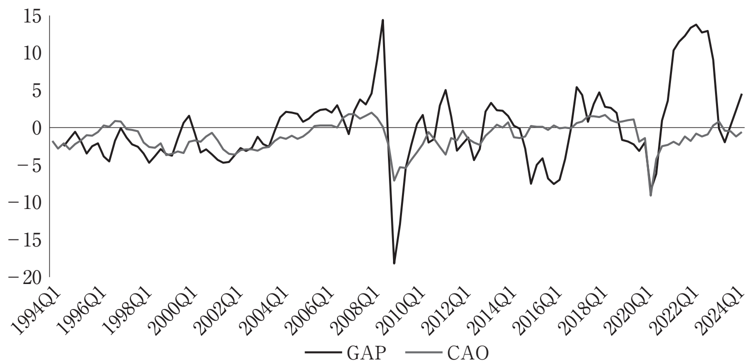
Variable	Coefficient	Std. Error	t-Statistic
g_{-1}	1.115715	0.08413	13.26181
g_{-2}	−0.429638	0.084116	−5.107711
R-squared	0.678322	Mean dependent var	−0.001962
Adjusted R-squared	0.675549	S.D. dependent var	0.051018
S.E. of regression	0.02906	Akaike info criterion	−4.222098
Sum squared resid	0.097961	Schwarz criterion	−4.175138
Log likelihood	251.1038	Hannan-Quinn criter.	−4.203031
Durbin-Watson stat	1.718547		

source: author's estimation

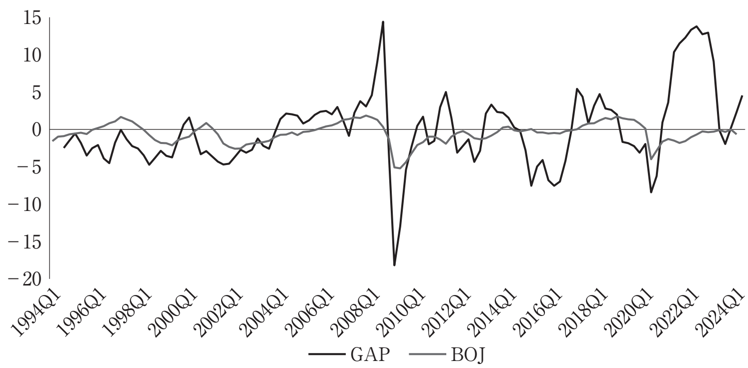
Chart 4: Estimation Result of Output Gap as percent of GDP

(1) Output Gap estimated with State Space Model

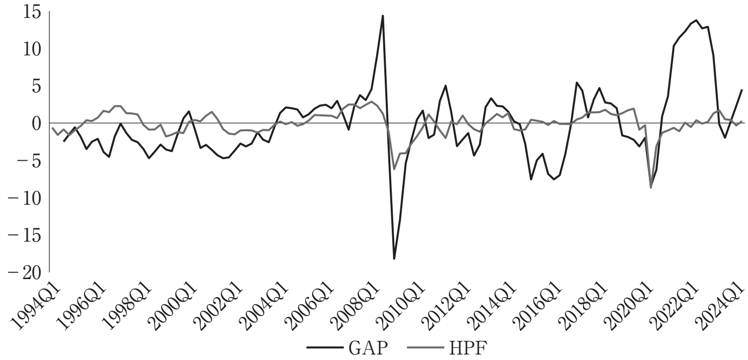
(2) Output Gap compared with CAO estimation



(3) Output Gap compared with BOJ estimation



(4) Output Gap compared with Hodrick-Prescott Filter



Source: author's estimation and each entity's data

5. Conclusion

Finally, two points must be focused on. One is estimation result and the other is on the stochastic process of the latent variables.

First, the results reported at Table 2 show very high significance of each estimation. However, the results of the study employing the state space model seem to overestimate in both direction of positive and negative compared with GDP gaps calculated by the CAO, the BOJ, and the Hodrick-Prescott filter. A lot of economists do not seem to agree that the output gap would exceed more than ten percent as the estimation results of the study. On the other hand, they succeed identifying the peaks and troughs of the output gap.

One of the most plausible reasons for the overestimation of the state space model is originated from the PPI inflation. The inflation rate reported at the Chart 1 (2) reveals that the PPI inflation is far larger than the CPI inflation. In December 2022, e.g., the PPI inflation exceeded 10 percent while the CPI inflation remained just 4percent. During the Great Recession after the Lehmann Brothers bankruptcy, on the other hand, the negative value of the PPI inflation is lower than that of the CPI. According to the data availability, the study adopts the PPI eliminating the consumption tax. This data selection causes somehow the overestimation.

Second, the study takes three assumptions for the latent variables: (1) the potential output follows a random walk process with a drift, (2) the drift also follows a random walk process, and (3) the output gap follows an autoregressive process of the first order. However, it is apparent that we have neither clear information on stochastic process nor lag order for the latent variables such as the potential output, the drift and the output gap. The study just adopts some very conservative assumptions from the existing studies including Watson (1986), Kuttner (1994) and so on.

These issues will be involved in the future studies.

(notes)

1) On deflation in general, see Kumar et al. (2003) and Nishizaki et al. (2014) describe deflation in Japan chronically. Decressin and Laxton (2009) also discuss the risk of deflation.

2) Jarociński and Lenza (2016) stress an inflation predicting role of GDP gap.

3) This paper was originally issued as a working paper of the Banco de España as Álvarez and Gómez-Loscos (2017).

4) Yoshioka (2002) also estimates the output gap of Malaysia, the Philippines, Singapore, and Thailand other than that of Indonesia.

5) In this paper, the authors of the National Bank of Poland officials explain the methodology of estimating output gap in Poland applying a vector error correction process while in Epstein and Macchiarelli (2010), the authors of the IMF staffs try to estimate Polish output gap adopting a production function approach.

6) This paper was originally issued as a working paper of the European Central Bank (ECB) as Morley et al. (2022).

7) The Bundesbank frequently refers the output gap in the outlook for the German economy in its monthly report. E.g., see Bundesbank (2023).

8) For Japanese production function, Kawamoto et al. (2017) assume Cobb-Douglas type.

9) Here, "potential labor hours" do not mean maximum possible labor hours. It is regarded as the same context as natural unemployment rate. Therefore, the actual labor hours can exceed the potential labor hours as same as the actual unemployment rate can go below the natural rate. The capacity utilization rate also possibly exceeds 100 percent.

10) See, e.g., Fig. 7.1 The 1880-1914 period: short-term interest rate, inflation, and real output on p. 327 of Taylor (1999)

11) Lienert and Gillmore (2015) suggest that the Reserve Bank of New Zealand now employs the production function approach to estimate the output gap.

12) Claus (1999) provides detailed information.

13) Gerlach and Smets (1999) also employ the same approach to measure output gap in Euro area and Kichian (1999) provides a practical estimates of this methodology in Canada.

14) Both Watson (1986) and Kuttner (1994) assume that the output gap series follow an autoregressive process by the second order.

15) Unlike (EQ-2), the output gap is defined multiplicatively with respect to the output in (EQ-17), whereas it is defined additively in (EQ-2). However, when taking logarithms, we could not find any significant difference between the additive and multiplicative definitions.

16) Surprisingly, the actual output is mathematically expressed in terms of output gaps and drifts.

17) One of the most practical reasons why the expectation of inflation is not assumed to follow further periods is due to multicollinearity problem.

18) Inflation rates are actually substituted with the first difference of logarithms. Because the following equations hold:

$$log(X_t) - log(X_{t-1}) = log\left(\frac{X_t}{X_{t-1}}\right) = log\left(\frac{\Delta X_t + X_{t-1}}{X_{t-1}}\right) = log\left(\frac{\Delta X_t}{X_{t-1}} - 1\right) \cong \frac{\Delta X_t}{X_{t-1}}$$

19) These data do not include services. Other than these data, however, it is impossible to eliminate the impact of consumption tax on prices.

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