Article

Identifying Trough of Recent COVID-19 Recession in Japan: An Application of Dynamic Factor Model

Shinji Yoshioka

Summary

In Japan, the Indexes of Business Conditions calculated by the Cabinet Office of the Government of Japan is widely employed for assessing business cycle and according to the discussion of the Committee for Business Cycle Indicators held on July 19th, 2022, the President of ESRI (Economic and Social Research Institute, Cabinet Office) has determined that a peak in business activities occurred in Japanese economy in October 2018 and a trough in May 2020. The Indexes of Business Conditions consists of three components, such as leading, coincident, and lagging indexes. The indexes are calculated by composing month-to-month percentage changes in multiple economic indicators. On contrary, in the U.S., for observing business condition, a stochastic business indicator is mainly employed. This study applies the latter U. S. approach estimating a latent stochastic business indicator for Japanese economy based on Stock and Watson (1989, 1990) using a dynamic factor model represented with a state space model and solved by Kalman filter. The estimated stochastic business indicator seems to fit quite well to existing Japanese official Indexes of Business Conditions. The estimated results appear to indicate that the trough month of the latest COVID-19 recession in Japan was May 2020.

Key words: Business cycle, Stochastic business indicator, COVID-19, Recession, Dynamic factor model, State space model, Kalman filter, Japan

JEL Classification: C13, C22, C43, C82, E32, and O53

1. Introduction

It is widely accepted that one of the most important roles of macroeconomic policies is to stabilize the economic fluctuations originated from the business cycle. The government is expected to manage its macroeconomic policies in a leaning-against-the-wind manner. Thus, the government and central bank authorities take an easing macroeconomic policy during a recession or slump, and some tightening policies will be required when the economy runs overheating. At the same time, the Government of Japan has been aiming for policy management based on EBPM (evidence-based policy making/management[1]). For these purposes, it is essential to have accurate understanding on the business cycle. For instance, the Government of Japan hiked its consumption tax rate from 8 percent to 10 in general in October 2019.[2] However, this was implemented during the recession.

Concerning to identification of business cycle turning points, this study focuses firstly on Indexes of Business Conditions of the Government of Japan, which is based on observable indicators, and secondly on stochastic approach, suggested by Stock and Watson (1989, 1990). The latter approach is also employed for many economic investigations: Melo et al. (2003) adopts for Colombian economy; Picchetti and Toledo (2002) estimate Brasirian industrial index; and Lemoine (2005) applies to the UK, French, German and the Euro-zone business cycles. Additionally, many other methodologies utilizing unobserved indicator approach are also explored: Hamilton (1989, 1990) introduces a Markov regime switching model; Kim and Nelson (1999) propose a Bayesian approach based on a Markov-switching model; Yoshioka (2009a) utilizes GDP gap estimated with a state space model for business cycle dating; and, Yoshioka (2009b) employs Markov Regime Switching model. Fukuda and Onodera (2001) also propose a new index of coincident economic indicators in Japan to improve the forecast performance. Related to business cycle dating, there are plenty of literatures using various methodologies, including Harding and Pagan (2002, 2006), Artis et al. (2004), and Chauvet and Hamilton (2006), which propose "quarterly real-time GDP based on recession probability index."

Before discussing on procedures and methodologies to calculate and estimate business cycle indicators, it appears very useful to confirm the business cycle dating in Japan. According to CAO (2016), at present, the reference date of a business cycle is first discussed in the Committee for Business Cycle Indicators, based on historical diffusion indexes, composed of all selected series of coincident diffusion indexes. Consecutively, the President of ESRI determines the reference date. The historical diffusion indexes determine the peak and trough for each selected time series of diffusion indexes (this is referred to as the individual turning point), which are calculated by marking the period from trough to peak with a plus, and the period from peak to trough with a minus. Since the change in direction is determined by smoothing irregular month-to-month movements of individual time series, the historical diffusion index calculated from these values is relatively smooth and reflects the basic movement of the business cycle. The last month when the historical diffusion index compiled from a selected series of coincident indexes stays below the 50-percent line corresponds to the cyclical trough; the last month when this index stays above the 50-percent line corresponds to the cyclical peak.

In addition, the peaks and troughs of each individual time series is dated by applying the Bry-Boschan method, which was developed by the U. S. National Bureau of Economic Research (NBER) and reported at Bry and Boschan (1971). In simple terms, this method determines the cyclical peak or trough by providing a series of rules. Two examples of this rule: that five months or more are required in the period between peak and trough, and that the duration of one cycle must be 15 months or more. This procedure, which also involves multiplication of the 12-month moving average, was presented along with a computer program to run. Table 1 reports the reference dates of business cycle in Japan that CAO (2022b) reveals.

Table 1: The Reference Dates of Business Cycles in Japan

Peak (By Month)	Trough (By Month)	Peak (By Quarter)	Trough (By Quarter)
Jun. 1951	Oct. 1951	2Q 1951	4Q 1951
Jan. 1954	Nov. 1954	1Q 1954	4Q 1954
Jun. 1957	Jun. 1958	2Q 1957	2Q 1958
Dec. 1961	Oct. 1962	4Q 1961	4Q 1962
Oct. 1964	Oct. 1965	4Q 1964	4Q 1965
Jul. 1970	Dec. 1971	3Q 1970	4Q 1971
Nov. 1973	Mar. 1975	4Q 1973	1Q 1975
Jan. 1977	Oct. 1977	1Q 1977	4Q 1977
Feb. 1980	Feb. 1983	1Q 1980	1Q 1983
Jun. 1985	Nov. 1986	2Q 1985	4Q 1986
Feb. 1991	Oct. 1993	1Q 1991	4Q 1993
May. 1997	Jan. 1999	2Q 1997	1Q 1999
Nov. 2000	Jan. 2002	4Q 2000	1Q 2002
Feb. 2008	Mar. 2009	1Q 2008	1Q 2009
Mar. 2012	Nov. 2012	1Q 2012	4Q 2012
Oct. 2018	May 2020	4Q 2018	2Q 2020

Source: CAO (2022b)

This study focuses on the business cycle in Japan and consists of four parts: the first and introduction section surveys business cycle dating; the second part describes the practical calculation process of the composite index of Indexes of Business Condition, officially adopted as reference series for assessing business cycles, and reveals methodologies to estimate procedure of stochastic business indicator presented by Stock and Watson (1989, 1990); the third part presents data, model and estimation results; and the final section briefly concludes this study. EViews V12 is employed for data processing and estimation, and Excel is utilized for drawing charts.

2. Methodology and Model

2.1 Methodology of Indexes of Business Conditions (Composite Index)

From April 2008 on, the Government of Japan has officially adopted a composite index as reference series for assessing business cycles, named Indexes of Business Conditions prepared by the Cabinet Office. This Indexes of Business Conditions (hereafter, CI) consists of three indexes, such as leading, coincident, and lagging indexes. Table 2 reports CI component.

Summarizing CAO (2009)[3], the Indexes of Business Conditions (CI) is calculated according to following four steps:

Step 1: A formula is used for calculating the symmetric percent change (r) of individual series (y) as in the following.

$$r_i(t)=200\times\frac{y_i(t)-y_i(t-1)}{y_i(t)+y_i(t-1)} \qquad \text{(EQ1)}$$

where r symmetric percent change
y individual series
i number assigned to each indicator
t time point

If the given time series is zero or a negative value, or is already in percentage form, simple arithmetic differences are calculated.

$$r_i(t)=y_i(t)-y_i(t-1) \qquad \text{(EQ2)}$$

Then, outliers (found only in the specific movement of each indicator as below) are replaced using the following step.

Step 1-1: The trend of individual series (mean percent change μ) is calculated by the 60-month backward moving average.

$$\mu_i(t)=\frac{\sum_{\tau=t-59}^{t} r_i(\tau)}{60} \qquad \text{(EQ3)}$$

where μ mean percent change

Table 2: Components of Indexes of Business Conditions

Leading Index	
	L1: Index of Producer's Inventory Ratio of Finished Goods (Final Demand Goods)
	L2: Index of Producer's Inventory Ratio of Finished Goods (Producer Goods For Mining and Manufacturing)
	L3: New Job offers (Excluding New School Graduates)
	L4: Machinery Orders at Constant Prices (Manufacturing)
	L5: Total Floor Area of New Housing Construction Started
	L6: Consumer Confidence Index
	L7: Nikkei Commodity Price Index (42items)
	L8: Money Stock (M2) (Change from Previous Year)
	L9: Stock Prices (TOPIX)
	L10: Index of Investment Climate (Manufacturing)
	L11: Sales Forecast DI of Small Business
Coincident Index	
	C1: Index of Industrial Production (Mining and Manufacturing)
	C2: Index of Producer's Shipments (Producer Goods for Mining and Manufacturing)
	C3: Index of Producer's Shipment of Durable Consumer Goods
	C4: Index of Non-Scheduled Worked Hours (Industries Covered)
	C5: Index of Producer's Shipment (Investment Goods Excluding Transport Equipments)
	C6: Retail Sales Value (Change from Previous Year)
	C7: Wholesale Sales Value (Change from Previous Year)
	C8: Operating Profits (All Industries)
	C9: Effective Job Offer Rate (Excluding New School Graduates)
Lagging Index	
	Lg1: Index of Tertiary Industry Activity (Business Services)
	Lg2: Index of Regular Workers Employment (Industries Covered) (Change from Previous Year)
	Lg3: Business Expenditures for New Plant and Equipment at Constant Prices (All Industries)
	Lg4: Living Expenditure (Workers' Households) (Change from Previous Year)
	Lg5: Corporation Tax Revenue
	Lg6: Unemployment Rate
	Lg7: Contractual Cash Earnings (Manufacturing)
	Lg8: Consumer Price Index (All items, Less Fresh Food) (Change from Previous Year)
	Lg9: Index of Producer's Inventory (Final Demand Goods)

Source: CAO (2018b)

Step 1-2: Percent change normalized by interquartile range (z) is calculated by applying the following formula.

$$z_i(t)=\frac{r_i(t)-\mu_i(t)}{Q3_i+Q1_i} \quad \text{(EQ4)}$$

where Q_1 the first quartile in the interquartile range of r

Q_3 the third quartile in the interquartile range of r

Step 1-3: Median of percent change normalized by interquartile range (z) is chosen for the common cyclical movement (ZC).

$$ZC(t)=Median\ of\ z_i(t) \quad \text{(EQ5)}$$

where ZC common cyclical movement of z

Step 1-4: The specific movement of each indicator (z') is calculated by subtracting the common cyclical movement from percent change normalized by interquartile range.

$$z_i(t)'=z_i(t)-ZC(t) \quad \text{(EQ6)}$$

where z' specific movement of each indicator z

Step 1-5: The symmetric percent change for the specific movement of each indicator (r') is calculated by adding up trend and the specific movement of each indicator multiplied by interquartile range.

$$r_i(t)'=z_i(t)'\times(Q3_i-Q1_i)+\mu_i(t) \quad \text{(EQ7)}$$

where r' specific movement of each indicator r

Step 1-6: The symmetric percent change for the common cyclical movement (rc) is calculated by multiplying the common cyclical movement by interquartile range.

$$rc_i(t)=ZC(t)'\times(Q3_i-Q1_i) \quad \text{(EQ8)}$$

Step 1-7: Outliers in the symmetric percent change for the specific movement of each indicator (r') are replaced using the following formula.

$$\phi_1(r_i(t)')=\begin{cases}-k'(Q3_i'-Q1_i') : r_i(t)'<-k'(Q3_i'-Q1_i') \\ r_i(t)' : -k'(Q3_i'-Q1_i')\leq r_i(t)'\leq k'(Q3_i'-Q1_i') \\ k'(Q3_i'-Q1_i') : k'(Q3_i'-Q1_i')<r_i(t)'\end{cases} \quad \text{(EQ9)}$$

where $Q1'$ the first quartile in the interquartile range of r'

$Q3'$ the third quartile in the interquartile range of r'

Then, the symmetric percent change for the common cyclical movement is added as follows:

$$\phi_2(r_i(t)') = \phi_1(r_i(t)') + rc_i(t) \quad \text{(EQ10)}$$

Step 2: Again, the trend of individual series (mean percent change μ') is calculated by the replaced 60-month backward moving average.

$$\mu_i(t)' = \frac{\sum_{\tau=t-59}^{t} \phi_2(r_i(\tau)')}{60} \quad \text{(EQ11)}$$

where μ' mean percent change

Percent change normalized by interquartile range (z'') is calculated by applying the following formula.

$$z_i(t)'' = \frac{\phi_1(r_i(t)') - \mu_i(t)'}{Q3_i - Q1_i} \quad \text{(EQ12)}$$

where z'' percent change normalized by interquartile range

Step 3: Composite percentage change (V) is calculated by adding up trend (composite mean percent change, $\bar{\mu}$) and the mean of percent change normalized by interquartile range (composite percent change normalized by interquartile range, $\bar{Z}$). In this process, composite percent change normalized by interquartile range is multiplied by the mean of interquartile ranges (composite interquartile range, $\overline{Q3-Q1}$) so that the levels of the trend component and the cyclical component coincide.

$$\bar{\mu}(t) = \frac{1}{n} \times \sum_i^n \mu_i(t)' \quad \text{(EQ13)}$$

$$\bar{Z}(t) = \frac{1}{n} \times \sum_i^n z_i(t)'' \quad \text{(EQ14)}$$

$$\overline{Q3-Q1} = \frac{1}{n} \times \sum_i^n (Q3_i - Q1_i) \quad \text{(EQ15)}$$

$$V(t) = \bar{\mu}(t) + \overline{Q3-Q1} + \bar{Z}(t) \quad \text{(EQ16)}$$

where $\bar{\mu}$ composite mean percent change

$\bar{Z}$ composite percent change normalized by interquartile range

$\overline{Q3-Q1}$ composite interquartile range

V composite percentage change

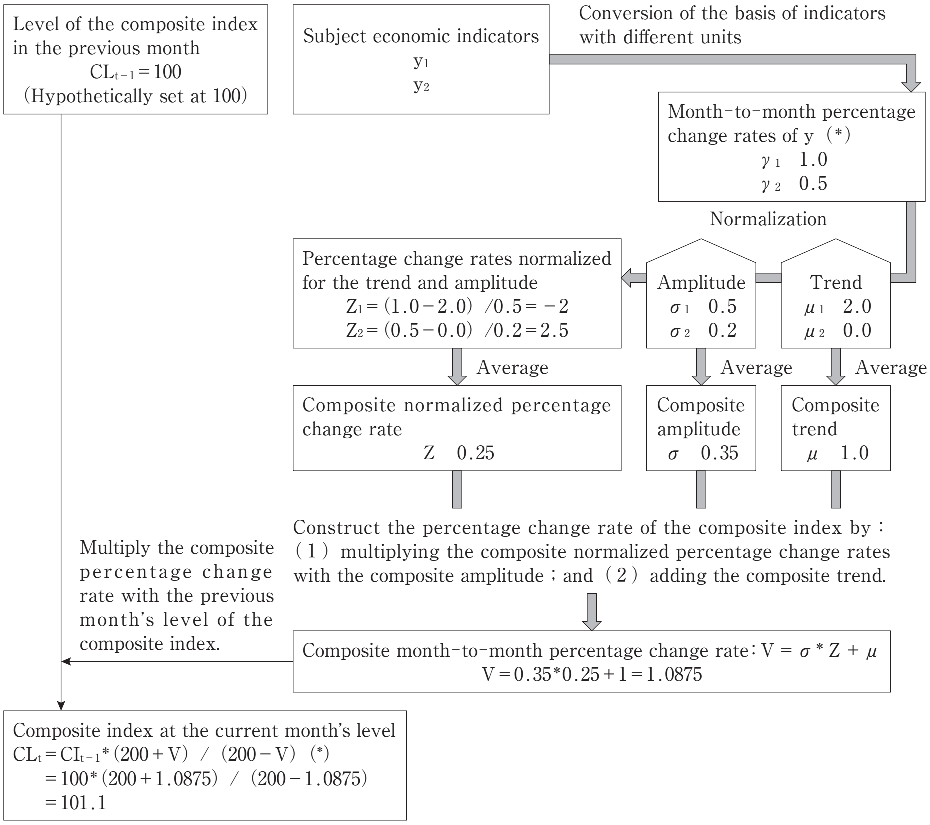
n number of indicators (y)

Step 4: Composite percentage change is cumulated.

$$I(t) = I(t-1) \times \frac{200 + V(t)}{200 - V(t)} \quad \text{(EQ17)}$$

$$CI(t) = \frac{I(t)}{I} \times 100 \quad \text{(EQ18)}$$

Chart 1: Composite index calculation flow and examples of values



Source: CAO (2009)

Finally, the index is rebased so that the value for the reference year is equal to 100. The current reference year is 2015.

These steps are conceptually illustrated at CAO (2009) as follow:

2.2 Model of Stochastic Business Indicator

Other than above business cycle indicators, including CI in Japan, Stock and Watson (1989, 1990) propose another type of stochastic business indicator that assumes a unique and latent index, affecting and revealing some observable indicators, such as production, employment, income, and consumption, etc. According to Stock and Watson (1989, 1990), assuming that this unique and latent index and the error terms follow autoregressive (AR) process, the model of stochastic business indicator is mathematically represented in the following model:

$$y_i(t)=\alpha_i+\beta_i c(t)+u_i(t) \quad \text{(EQ19)}$$

$$c(t)=\gamma+\tau_1 c(t-1)+\tau_2 c(t-2)+\cdots+\tau_n c(t-n)+e(t) \quad \text{(EQ20)}$$

$$u_i(t)=\lambda_i u_i(t-1)+\lambda_i u_i(t-2)+\cdots+\lambda_i u_i(t-m)+\varepsilon_i(t) \quad \text{(EQ21)}$$

where	y	Observable business indicators (i=1, 2, 3, …)
	c	Unique and latent business indicator
	u, e, ε	Error term (i=1, 2, 3, …)
	i	Number of observable indicators
	n	Number of lags of AR process for c
	m	Number of lags of AR process for u
	α, β, γ, τ, λ	Parameters

Using lag operator L, above model can be expressed as follows:

$$y_i(t)=\alpha_i+\beta_i c(t)+u_i(t) \quad \text{(EQ22)}$$

$$\varphi(L)c(t)=\omega+e(t) \quad \text{(EQ23)}$$

$$\theta(L)u(t)=\varepsilon(t) \quad \text{(EQ24)}$$

Here, φ in (EQ23) represents a lag polynomial of $\varphi=1-\varphi_1 L-\varphi_2 L^2-\cdots-\varphi_n L^n$ and θ in (EQ24) does that of $\theta=1-\theta_1 L-\theta_2 L^2-\cdots-\theta_m L^m$, respectively. On the other hand, error term e in (EQ23) is a scalar stochastic variable that follows $e \sim N\ (O, \sigma^2)$, and ε in (EQ24) is too a scalar stochastic variable that follows $\varepsilon \sim N\ (O, \sigma^2 H)$.[4)]

Since this model for stochastic business indicator includes latent variables, the equation system is represented as a state space model. According to Okusa (1992), the generalized state space representation of the stochastic business indicator is as follows:

1) **State variable δ**

$$\delta=\begin{bmatrix} c(t) \\ c(t-1) \\ \vdots \\ c(t-n+1) \\ u(t) \\ u(t-1) \\ \vdots \\ u(t-m+1) \end{bmatrix} \quad \text{(EQ25)}$$

2) **Observation equation y**

$$y(t)=Z\delta(t) \quad \text{(EQ26)}$$

3) **Transit equation δ**

$$\delta(t)=X\delta(t-1)+\xi(t) \quad \text{(EQ27)}$$

4) **Disturbance term ξ**

$$\xi(t)\sim N(O_{i+1},\ \sigma^2\Sigma) \quad \text{(EQ28)}$$

where $Z=[\beta \quad O_{i,n} \quad I_i \quad O_{i,m}]$ (EQ29)

$$X=\begin{bmatrix} \begin{matrix} \varphi_1 & \varphi_2 & \cdots & \varphi_{n-1} & \varphi_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \end{matrix} & O_{n,im} \\ O_{im,n} & \begin{matrix} \Theta_1 & \Theta_2 & \cdots & \Theta_{m-1} & \Theta_m \\ I_i & O_i & \cdots & O_i & O_i \\ O_i & I_i & \cdots & O_i & O_i \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ O_i & O_i & O_i & I_i & O_i \end{matrix} \end{bmatrix} \quad \text{(EQ30)}$$

Of course, the state variable δ in (EQ27) is an $n+m$ vector. According to usual definition, I_k in (EQ29) and (EQ30) means a unit matrix with k rows and columns, and $O_{k,l}$ in (EQ28), (EQ29) and (EQ30) represents a null matrix with k rows and l columns. And, Σ in (EQ28) means a diagonal matrix with its elements of, $\Sigma=diag(1\ O'_{n-1}\ h_1\ h_2 \cdots h_{n-1}\ h_n\ O'_{(n-1)m})$, while h is a diagonal element of H. ϕ is a parameter of n-degree lag polynomial for the latent index. Θ in (EQ30) is a diagonal matrix with elements of θ, which is a parameter of m-degree lag polynomial for the error term u in (EQ22) and (EQ25).

Here, it should be strongly stressed that both stochastic business indicator and error terms are unobserved. This means that we do not have enough information about the generating processes of those series. For instance, Hamilton (1994) employ VAR(1) process while Harvey (1993) adopts ARMA (1,1) process. At the same time, the lag orders are also unknown.[5] In order to avoid these difficulties, this study takes following three assumptions, which seem adequately plausible, to simplify the model according to existing literatures, including Stock and Watson (1989, 1990), Okusa (1992) and Yoshioka (2010):

1) The observable indicators are taken from production, employment, income, and consumption, i.e., $i=4$.
2) The unique and latent business indicator c follows AR(2) process, i.e., $n=2$.
3) The error term u follows AR(1) process, i.e., $m=1$.

Above model for stochastic business indicator will be transformed into following simplified state space model system consisting of (EQ31) and (EQ32):

1) **Observation Equations**

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \beta_1 & 1 & 0 & 0 & 0 \\ \beta_2 & 0 & 1 & 0 & 0 \\ \beta_3 & 0 & 0 & 1 & 0 \\ \beta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c(t) \\ u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} \quad \text{(EQ31)}$$

2) **Transit Equations**

$$\begin{bmatrix} c(t) \\ u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 \end{bmatrix} \begin{bmatrix} c(t-1) \\ c(t-2) \\ e(t) \\ u_1(t-1) \\ u_2(t-1) \\ u_3(t-1) \\ u_4(t-1) \end{bmatrix} \qquad \text{(EQ32)}$$

This simplified state space model will be solved with Kalman filter presented at Kalman (1960). In this study, further explanation for a state space models and Kalman filter will be out of target. For comprehensive information on application of state space model to econometric field, Harvey (1981) is one of the most useful literatures, if necessary. Apart from Kalman's original paper, Meinhold and Singpurwalla (1983), Snyder and Forbes (1999), and Grewal and Andrews (2002) will provide further information on Kalman filter and its algorithm.

3. Data and Estimation Results

According to the assumption and the model presented in the previous section, following actual and observable data are employed[6]:

1) **Production**: Index of Industrial Production (Mining and Manufacturing) published by the Ministry of Economy, Trade and Industry, seasonally adjusted series.

2) **Employment**: Index of Non-scheduled Hours Worked in Monthly Labor Survey published by the Ministry of Health, Labor and Welfare, seasonally adjusted series.

3) **Income**: Real Wage Index of Total Cash Earnings (establishments with five employees or more) in Monthly Labor Survey published by the Ministry of Health, Labor and Welfare, seasonally adjusted series.

4) **Consumption**: Retail Commercial Sales Value of Monthly Report on the Current Survey of Commerce published by the Ministry of Economics, Trade and Industry, deflated to real term by Consumer Price Index published by the Statistics Bureau of Japan, and seasonally adjusted by X-12 with a default option by author.[7)8)]

This study utilizes data available on August 10, 2022. Table 3 reports data descriptions while Charts 2 depict data.

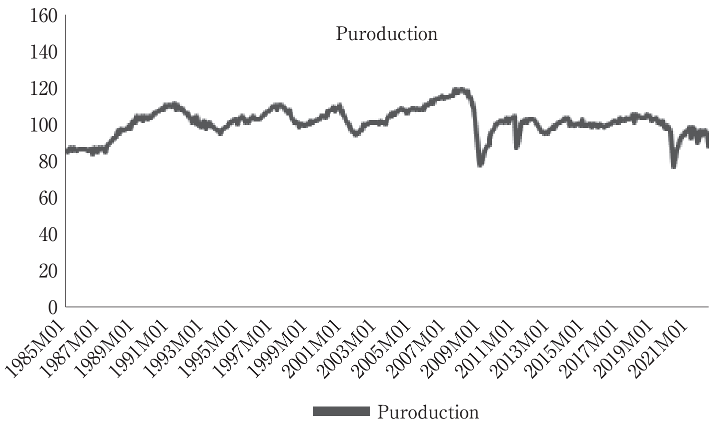
First of all, augmented Dickey-Fuller (ADF) unit root tests based on Dickey and Fuller (1979, 1981) are executed in order to check the data generating process of relevant above four data series. Table 4 reports the results. These results strongly suggest that log

Table 3: Data Descriptions

	Production	Employment	Income	Consumption
Mean	101.3891	118.2325	105.9465	120.083
Median	101.6	116.4	106.2	119.9197
Maximum	119.4	151.1	118.2	141.3631
Minimum	77.2	83.2	95.1	101.3818
Standard Deviation	7.520537	12.7776	4.204307	6.013581
Skewness	−0.404804	0.799901	−0.188061	−0.449092
Kurtosis	3.544494	3.238218	2.349769	4.057315
Observations	449	449	449	449
Sample Period	Jan. 1985- May 2022	Jan. 1985- May 2022	Jan. 1985- May 2022	Jan. 1985- May 2022

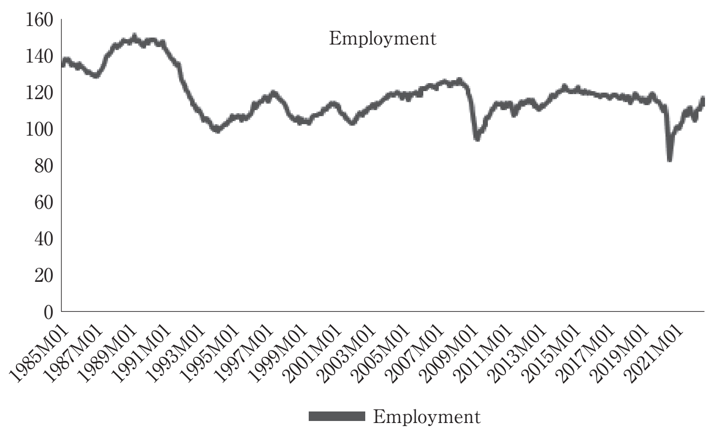
Source: Author's calculation

Chart 2: (1) Production



Source: Ministry of Economy, Trade and Industry

(2) Employment

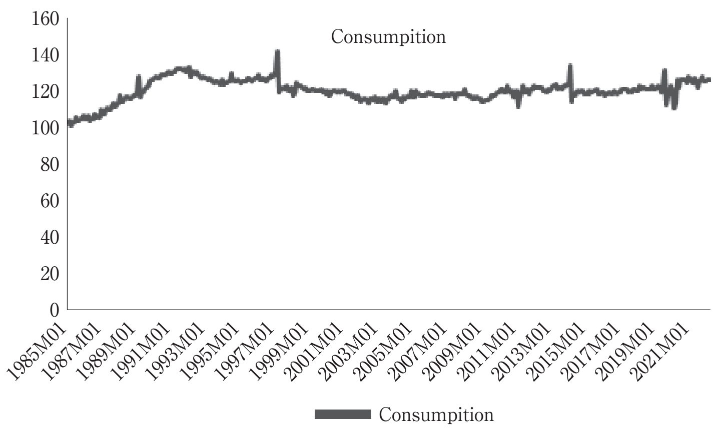


Source: Ministry of Health, Labor and Welfare

(3) Income

Source: Ministry of Health, Labor and Welfare

(4) Consumption



Source: Author's calculation based on statistics of the Ministry of Economy, Trade and Industry and the Statistics Bureau of Japan

difference data must be employed for estimation because log level data cannot reject the unit root.

Based on above model and data, the unique and latent business indicator is estimated.[9] Table 5 reports the estimation results. The coefficient for production is relatively large, while those for income and consumption are small. This fact may indicate the sensitivity to business cycle, of course. Among those, this study cites the Monthly Labor Survey for wage data, which are known to had been improperly processed.[10] It is possible that the unreliable statistics may have some influences. Regarding the explanatory power of consumption, it should be pointed out that the introduction of the consumption tax in 1989 and the subsequent tax rate hike caused sizable disturbance on consumption and may have reduced some impact on business cycle.

Table 4: Results of ADF Tests

	log level			log difference		
	t-statistics	p-value	lag	t-statistics	p-value	lag
Production	−3.26957	0.0727	1	−19.70992	0	0
Employment	−2.285967	0.4403	2	−14.07262	0	1
Income	−3.127008	0.1013	5	−15.365	0	4
Consumption	−2.795128	0.1998	8	−18.70258	0	2

Note: (1) Lag length are decided according to Akaike Information Criteria developed by Akaike (1969, 1973) under condition of maximum 16 months.
(2) p-values are calculated at a one-sided basis on MacKinnon (1996).
Source: Author's calculation

Table 5: Estimation Results

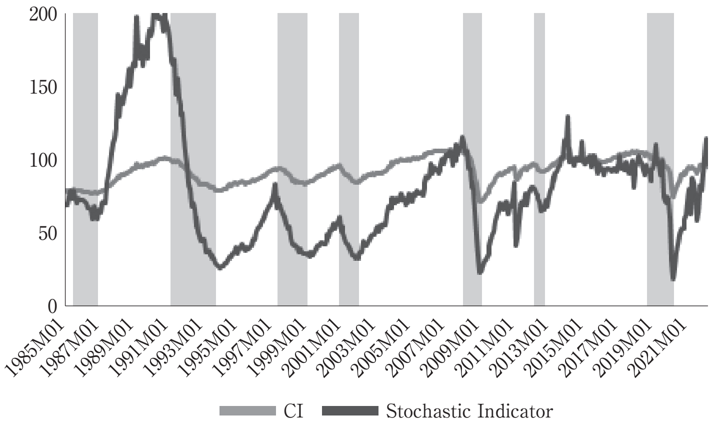
		coefficient	std. error	t-statistics	R^2 adjusted
Production	constant	−9.70E-05	0.00034	−0.28503	0.889385
	stochastic indicator	0.207966	0.003469	59.95034	
Employment	constant	−0.000481	0.000565	−0.852577	0.475463
	stochastic indicator	0.115826	0.005754	20.1291	
Income	constant	8.53E-05	0.000622	0.137078	0.012706
	stochastic indicator	0.015215	0.006344	2.398492	
Consumption	constant	0.000385	0.000903	0.426684	0.180519
	stochastic indicator	0.091307	0.009201	9.923073	

Source: Author's calculation

Finally, Chart 3 depicts the estimation results compared with the coincident composite indicator of the Indexes of Business Conditions (CI) calculated by the Government of Japan. The estimated stochastic business indicator clearly shows the peak and trough of business cycle. In particular, it indicates sharper decline during 1985-86 recession than the CI of the Indexes of Business Conditions. However, On the other hand, the estimated stochastic business indicator seems to move too large.[11] At the peak of the bubble economy in the early 1990s, the level of the estimated stochastic business indicator reached at approximate 200, nearly double the level of the CI. On contrary, the trough of babble-collapsed recession in mid-1990s indicated less than thirty.

Here, Table 6 compares three sets of peak and trough months of Japan's business cycle, identified by (1) the official reference dates of the Government of Japan, reported at CAO (2022b), (2) turning points of the Indexes of Business Conditions (CI), and (3) those of the estimated stochastic business indicator in this study. They are not necessarily coincident, but sizably close to each other. Among those, it should be stressed that the estimated stochastic business indicator accurately captures the extraordinary economic decline caused by the disaster of the Great East Japan Earthquake while three indicators coincide the

Chart 3: Stochastic Indicator and Index of Business Conditions (CI) (2015 = 100)



Source: Cabinet Office data and author's calculation
Note: Shadowed periods indicate recessions.

Table 6: Peak and Trough Months of Business Cycle in Japan

		Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
(1) CAO (2022a)		Jun. 1985	Nov. 1986	Feb. 1991	Oct. 1993	May 1997	Jan. 1999	Nov. 2000	Jan. 2002
(2) CI		May 1985	Nov. 1986	Oct. 1990	Dec. 1993	May 1997	Dec. 1998	Dec. 2000	Dec. 2001
(3) This Study		May 1985	Nov. 1986	Oct. 1990	Jan. 1994	Mar. 1997	Feb. 1999	Dec. 2000	Nov. 2001
difference	with CAO	−1	0	−4	+3	−2	+1	+1	−1
	with CI	0	0	−4	+1	−2	+2	+1	−2
		Peak	Trough	Peak	Trough	Peak	Trough		
(1) CAO (2022a)		Feb. 2008	Mar. 2009	Mar. 2012	Nov. 2012	Oct. 2018	May 2020		
(2) CI		Oct. 2007	Mar. 2009	Mar. 2012	Nov. 2012	Oct. 2018	May 2020		
(3) This Study		Feb. 2008	Apr. 2009	Mar. 2012	Nov. 2012	May 2019	May 2020		
difference	with CAO	0	+1	0	0	+7	0		
	with CI	+4	+1	0	0	+7	0		

Note: The number of difference rows counts the difference between turning points of the estimated indicator of this study with CAO (2022a) and CI. If it's positive, it's preceding, and if it's negative, it's lagging. A zero indicates a match.
Source: CAO (2022b), Cabinet Office data, and author's calculation

trough of the latest COVID-19 recession. i.e., the estimated stochastic business indicator identifies May 2020 as the trough of the latest recession, which appear quite plausible and acceptable among Japanese economists.

During the estimation period of 1985–2022, fourteen turning points, i.e., seven peaks and seven troughs, are observed. Among those, the estimated indicator of this study precedes the CAO's official dates four times and lags five times while matching five times. At the same time, it precedes the CI's turning points three times and lags six times while matching five times. The largest difference with both CAO (2022b) and CI is seven months. The performance of the stochastic business indicator estimated in this study does not seem so bad, especially, identifying the turning points of business cycle. Moreover, all of three indicators, i. e., the official dating of the Government, CI, and the estimated stochastic business indicator of this study coincide identifying the trough of COVID-19 recession that occurred in May 2020.

4. Conclusion

This study has successfully estimated the stochastic business indicator. In particular, the estimated indicator shows sizably significant performance in identification of business cycle turning points. However, there are also some remaining issues: one is estimated result and the other is methodology. While the estimated indicator succeeds identifying the turning points of business cycle, it apparently fails to represent volume of economic movements. In this case, we should regard that the sense of economic volume is measured by sum of aggregated value added, but it does not make sense that the total value added of the Japanese economy fell below half before and after the collapse of the bubble economy as the results suggest. Another improvement is required for the estimation method. According to existing literature including Stock and Watson (1989, 1990), Okusa (1992), and Yoshioka (2010), this study adopts very conservative assumptions for model specification: 1) the observable indicators are taken from production, employment, income, and consumption; 2) The unique and latent business indicator follows AR(2) process; and the error term follows AR(1) process. On the other hand, there is no clear information about stochastic process and lag order that the latent variables and error terms follow in a state space model. In addition, this study employs Kalman filter for solving the model, However, Kalman filter introduced by Kalman (1960) seems rather old algorism and De Jong (1988, 1991) and De Jong and Chu-Chun-Lin (1994) have proposed some new solution algorism. This study cannot include these new approaches. These issues will be adopted in the future research.

Notes

1) For detail about EBPM, see U. K. Cabinet Office (1999), and so on. In case of the Government of Japan, Inada (2022) summarizes and evaluates its current development of EBPM in Japan focusing on logic model.

2) This consumption tax rate hike includes some exceptional items as reduced tax rates. For more detailed information, see ministry of Finance (2019).

3) Yoshizoe et al. (2003) provide more detailed information and discussion.

4) Of course, O is a null matrix.

5) These points will be discussed later at the final section of conclusion.

6) All data are monthly.

7) The Statistics Bureau of Japan releases the index of real consumption expenditure in Family Income and Expenditure Survey. But this series of data starts in 2000. For the reason of data availability, this series cannot be adopted.

8) As well-known, the Census Burau of the United States has already released X-13 as a seasonal adjustment tool that has took over X-12. However, CAO (2018a) reveals that X-12 is employed for calculating the Indexes of Business Conditions.

9) For long, there has been criticism that production has too much influence on the Indexes of Business Conditions, and the CAO (2022a) has announced that they will start considering the introduction of a new indicator.

10) For detail about this incident, see the press release of the Ministry of Health, Labor and Welfare (2019).

11) Yoshioka (2010) addresses the same characteristic feature of this type of stochastic indicator.

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