論説

Economic Benefit of Temporary Migration with Firm Heterogeneity

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Abstract:

The paper uses a monopolistic competition model with heterogeneous firms and productivity asymmetry to evaluate the effects of temporary movement of labour from a developing to a developed country. In autarky, migration induces a decrease of survival productivity threshold implying that less productive firms of the receiving country are now able to survive. The receiving country also enjoys an increase in the welfare because of decrease of the price level. However the people living in the source country experience a decrease in welfare when remittances are not taken into consideration. In open economy the exporting firms of the receiving and domestic firms of sending countries experience a fall in minimum required productivity threshold. The result on the welfare and price level of the sending and receiving countries are ambiguous.

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1. Introduction

Movement of labour across national borders is now a core element of public and policy debates. The debates however focuses mainly on immigration and temporary migration of labour and the effects of temporary migration receive less attention. The economic literature on migration also heavily biased in analysing the effects of permanent migration and ignores of the case of temporary migration (Dustmann and Görlach 2016). In this paper we aim to address this by issue that is how temporary movement of labour can

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bring benefits to the countries concerned.

Many countries now control temporary migration of labour through bilateral migration agreements. Temporary migration of labour is additionally addressed under the framework proposed by World Trade Organization (WTO) through the Mode 4 of the General Agreement on Trade in Services (GATS). So far Economic literature analysed the benefit of GATS type labour migration from a global perspective. Winters (2002) estimated that increased mobility equivalent to 3% of the receiving countries’ work forces would generate $156 billion per year in extra economic welfare. Rodrik (2004) pointed that the highest benefit of liberalisation at present world can be generated by the liberalisation of movement of temporary workers and not from much discussed increased market access or liberalisation of agricultural trade. However negotiations on the issue of migration are problematic as countries are in general despite the potential for huge benefit, still reluctant to open up their borders for movements of labour under WTO (Winters 2005). The growing public sentiment against migration is also evident inpopularity of anti-globalization and anti-establishment movements in Western political arenas. Trade negotiators and politicians of developed countries are uneasy about the issue of labour movement because of these political pressures. It is often argued that the sending countries cannot guarantee return and temporariness (Friedman and Ahmed 2008), hence there is a resistance to open up borders within GATS framework and this resistance is likely to be continued (Poot and Strutt 2010). The reluctance and resistance are even notable in bilateral migration agreements.

It can be noted that the countries involved in international negotiation are lacking proper understanding of how temporary movement of people can be beneficial. What is missing here are some studies aiming to evaluate the benefit of movement of labour from a single country’s perspective. If a worldwide movement of labour is initiated, studies like Winters (2002), Rodrick (2004) will be applicable but these are not useful to provide policy guidelines to a single country in migration negotiations rounds. One study by Walmsey and Winter (2007) divided the world in separate regions and gains are not evenly distributed but again the study is not useful to provide guideline from a single country’s perspective. Schiff (2007) and Bchir (2008) addressed the Mode 4 trade but only in the context of receiving countries illegal migration problem. Schiff (2007) studied optimal migration policy under permanent, guest-worker (GW) and Mode 4 program. In summary the paper finds that policy makers should consider implementing Mode 4 rather than Guest Workers’ Program as the number of illegal migrants is lower, number of migrants is higher and government’s welfare is higher with Mode 4.

In general, there a dearth of literature on temporary migration which can give guidelines to the policy makers in international migration negotiations. A large body of literature addressed the issue of wage determination of immigrants and its impact on native’s
earning (for a review of literature, Kerr and Kerr 2011). These works assume that immigrants compete with natives in the labour market for wage and employment opportunities. But bilateral migration agreements do not cover permanent migration and the wage determination process of temporary workers is not necessarily the same as the wage determination process of natives and immigrants. Temporary workers may not even compete with the domestic workers in labour market. One of the oldest successful contractual labour migration programs is Canada’s ‘Seasonal Agricultural Workers’ Program’ initiated in 1960s. The reason of the initiation of the program was the unavailability of agricultural workers in the pick season even at a high market wage (Verma 2004).

Moreover, the temporary workers can be contracted to received wage rate lower than the prevailing market wage rate. They can differ according to nationality. For example wages of housemaids in Gulf countries are Philippines Dh 1,470 ($400), India Dh 1,100, Sri Lanka Dh 825, Bangladesh Dh. 750 (Gulf News 2008). A recent body of literature has however started to pay more attention to the issue of temporary migration though not necessarily on wage determinations process. Notably Dustmann and Görlach (2016) provides a survey of literature and analyses the possible impact of migration temporariness to the both host and source countries. Dustmann and Mestres (2010) look at the relationship between temporariness and remittances and identified that temporary migrations are likely to lead to higher remittances flow.

In this regards the brain drain (gain) literature deserves some attention. The aim of the literature is to look at the gain or loss of the developing countries through outflow of human capital (for reviews see Chowdhury and Telli 2016, Docquier and Rapoport 2012; Schiff 2006). But they are in general applicable to skilled labour movement whereas bilateral or multilateral agreements look at both skilled and unskilled movements. The brain drain (gain) studies evaluate the cost of migration on unilateral basis and not much concerned about the role of remittances in the economies of sending countries.

The paper addresses the gap in the literature to evaluate the economic benefit of temporary movement from a single country’s perspective. It incorporates international migration within a Melitz type (Melitz 2003; Helpman et al. 2004) intra-industry trade model, that is, in a monopolistic competition model where firms’ productivities are allowed to be heterogeneous. Falvey et al. (2006, 2005 and 2011) extended the model of heterogeneous firm to allow for inter-country asymmetry of productivity and size. We have incorporated migration and remittances in the model. We utilised these models in our paper as simultaneous presence of intra-industry trade between technologically advanced-backward countries and international migration is observed in many parts of the world. The interlinkage of migration and trade was also analysed by Iranzo and Peri (2009) and Egger et al. (2012) though not specifically focusing on temporary migration.

Following the tradition of the literature of firm heterogeneity we analyse the effects of
migration in both autarky and in open economy. In autarky, the effects of international migration on the welfare of technologically advanced country are identified clearly. International migration, in autarky, allows less productive firms of the receiving countries to survive in the market. The welfare of the country is higher as price level falls because of migration. However the technologically backward country that is from which migration takes place becomes worse off as the welfare of the people who remain behind fall. This result is contradicting with the results of highly influential paper of Grubel and Scott (1966) that showed that welfare of those who stay behind remains unchanged. However if we allow for remittances then the welfare of the sending country may increase but it is not a certainty. In open economy the results are not so straight forward. The exporter and domestic firms of both receiving and sending countries may experience decrease in survival productivities. The price equations also do not show clear pattern for improvement of welfare.

The organisation of remaining part of the paper is as follows. The second section proposes the basic features, assumptions and notations of the model. In the third section we look at the equilibrium in autarky with migration. In forth section, we assume open economies and see how the migration has an effect on a country’s trade, productivity and welfare. The fifth section concludes the paper.

2. The Model, Basic Assumptions and Notations

2.1 Demand, Wage rate of Host and Source Country

Following Falvey et al. (2006, 2005 and 2011) we assume two countries defined as ‘Host’ and ‘Source’ consisting of two sectors of production. In host country, one of the sectors is a homogenous and perfectly competitive sector which we denote as \(H\). The other sector is consisted of differentiated goods and denoted by \(Y\). Preference of the representative consumer is Cobb-Douglas type defined by the following utility function:

\[
U = H^{1-\beta} Y^\beta
\]

where \(H\) = Homogenous goods
\(Y\) = Differentiated goods
and \(0 < \beta < 1\)

The total expenditure is given by \(H + PY\) where the relative price of differentiated good is given by \(P\). The price of homogenous good is set as numeraire and assumed equal to 1. As the homogenous sector is perfectly competitive we get:

\[
w_H = 1
\]
where $a_{H}$ is the fixed unit labour coefficient implying the amount of labour required to produce one unit of homogenous good and $w$ is the wage rate. The equation shows that in a competitive sector, unit price of a product is equal to unit production cost of the product. From the equation (2), the wage rate of homogenous sector is obtained as $w = 1/a_{H}$. As domestic labour can move freely between homogenous and differentiated sectors, $w$ is the equilibrium wage rate of domestic labour in home in both sectors. The employment in homogenous sector is given as:

$$L_{H} = a_{H}H$$  \hspace{1cm} (3)

where $L_{H}$ is the total labour used in homogenous sector. Assume that the total endowment of domestic labour is given by $\bar{L}$. $M$ is the amount of migrant labour. Hence,

$$\bar{L} + M = L_{H} + L_{Y}$$  \hspace{1cm} (4)

where, $L_{Y}$=labour in differentiated sector. Let us assume that the market size of the host country is given as $\Phi$ which shows the total expenditure within the economy. Then we must have,

$$H + PY = \Phi$$  \hspace{1cm} (5)

Forming lagriagian and using first order conditions, the solutions for equilibrium demands of homogenous and differentiated goods are obtained as

$$H = (1 - \beta)\Phi$$ and $$Y = \frac{\beta \Phi}{P}$$  \hspace{1cm} (6)

The economy of the source country is defined in similar fashion. The complete description of source country is given later.

2.2 Defining Migration

Assume that the country can import additional labour from abroad through bilateral or multilateral agreements. The foreign workers work for only one period and then return back with the income they receive. In autarky they can be regarded as returning back with income in term of goods. The migrant workers receive wage $(w-s)$ where $0 \leq s < w$.

Hence migrant workers are recruited at a lower wage rate compared to the wage rate of domestic workers. This idea of modelling migrants’ wage came from Chanda (2001) who suggested a few mechanisms to make GATS workable. It is sometimes thought that wage rates of migrants should be at par with the natives’ wage rates. Within GATS format wage differentials actually give the poor countries the advantage to export labour to rich countries. To make it simple we can just consider it as the price of labour service instead of wage, thus the labour service can be acquired from any part of the world and the low
cost one is the most desired. However, as noted by Chanda, lower reservation wage rate of migrants exposes them to higher possibility of exploitation. Additionally, as the wage rates are low the producers may only recruit migrant workers thereby it may have adverse impact on employment opportunities of native workers. Thus a balance should be found between these conflicting objectives to make agreements for temporary migration workable.

We may also want to find empirical evidence of whether in reality migrants receive lower wage compared to the natives. Such studies are not readily available. It can be however established using simple deductive reasoning. The empirical works on immigration showed that immigrants in general start with a lower income than that of comparable group of natives (Borjas 1994). The studies have shown that after a certain period of time (e.g. 20 years) immigrants may experience convergence of income. Thus even with the convergence, the life time income of immigrants is lower than the comparable group of natives.

It should be here noted that in this paper we are not concerned about immigrants or permanent migrants. Rather the focus is on temporary migrants. Temporary migrants wage determination process will depend on the relative bargaining power of the country of origin. Different cohorts of temporary workers may experience different wage rates. For example, migrants in the Middle East receive different wage rates based on their nationalities. To what extent the wage rate reflects their intrinsic productivity is still an open question. The migrants of poor countries can offer labour services at a lower wage rate because of low reservation level. The lower wage rates also do not necessarily show discrimination, such as gender or racial discrimination prevailed in history.

We assume in this paper that the migrants cannot replace domestic labour given the regulation that domestic labour must be fully employed before employing foreign labour. The entry of migration is limited by a prefixed quota. This assumption highly reflects the labour recruitment strategy of the developed countries. But such quota is often practised because of the socio-economic concerns of adverse effects of migration. We have not modelled any such adverse effects. The assumption of restrictive quota nevertheless helps us to concentrate on the main goal of assessing the benefit of temporary movement of labour.

2.3 Demand, Production and Labour Allocation in Differentiated Sector

2.3.1 Demand of Differentiated Goods

In the section 2.1 we have derived the equilibrium demand of homogenous and differentiated goods. In this section we look at the demand of $Y$ more closely as it is defined as a composite index of differentiated goods. This composite good consist of a bundle of closely related product varieties which are close but imperfect substitutes. The
consumption of $Y$ is given by following standard Dixit-Stiglitz’s ‘love of variety’ form:

$$Y = \left[ \int_{i \in V} y(i)^{\epsilon} di \right]^{1/\epsilon}$$

where, $y(i)$ stands for variety $i$ and $V$ stands for available set of varieties, $0 < \rho < 1$. Elasticity of substitution is defined as $\varepsilon = \frac{1}{1-\rho} > 1$. The higher is $\varepsilon$ the better substitute the varieties are. This formulation captures the notion of preference for diversity as the consumer prefer to spread consumption over the spectrum of differentiated good rather than concentrating on single variety. The total demand of differentiated good has been obtained as $Y = \frac{\beta \Phi}{P}$. Therefore total expenditure on differentiated good is $PY = \beta \Phi$. The budget constraint for consumption of differentiated good is then,

$$\int_{i \in V} p(i)y(i)di = PY = \beta \Phi$$

Using Lagrange method, the solution obtained via the two stage budgeting procedure is,

$$y(i) = Ap(i)^{-\varepsilon}$$

where $A = \beta \Phi P^{\varepsilon-1}$, $\varepsilon = \frac{1}{1-\rho}$ and $P = \left[ \int_{i \in V} p(i)^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}$

Hence, $P$ denotes the aggregate price index. The solution shows the demand for individual variety $i$ is increasing in the aggregate price index and the market size. Additionally, the demand is negatively related with the own price. The higher is the $\varepsilon$ or elasticity of substitution the higher is the aggregate price index and the own price.

### 2.3.2 Production and Wage Rate in Differentiated Sector

Firms incur two types of costs in production of differentiated goods. One is a constant marginal cost of production, denoted by $a$, which is different across firms. The other one is fixed cost $F_0$ in production which is identical across all firms and all countries. For example, labour requirement $L(i)$ of the firm producing variety $y(i)$ with marginal cost $a$ is given as $L(i) = F_0 + ay(i)$. This specification shows increasing returns to scale as average cost declines with output. It is assumed that both fixed cost and marginal costs are exogenous to the model.

In the labour market, domestic workers are recruited at the wage rate $w$. Firms can employ migrants from outside after full employment of domestic labour. As previously discussed, migrants are recruited only by firms operating in differentiated sector. It is also assumed that $M$ number of migrants are recruited at a wage rate $(w-s)$. $M$ shows entry quota which is assumed to be sufficiently low so that a number domestic labour always
work in differentiated sector. As the number of domestic labour in sector $Y$ is given as $\bar{L} - L_H$ the average wage rate in the differentiated sector is calculated as,

$$W = w - \frac{sM}{L_Y}$$

(8)

In order to calculate the average wage we have made a crucial assumption that $M$ gives the pool of migrant labour in the economy. Note that full employment of domestic workers must be ensured, therefore if firms are regulated such that migrants can only be hired after the full employment of natives, then it provides firms an incentive to delay the entry as wage rate of migrants is lower than the wage rate of the natives. All firms then would try to enter after full employment the native labour. We therefore assume that total migrants are given by a pool. A firm may get the allocation of labour by a mechanism similar to a lottery. All firms would pay the average wage for per unit of labour recruited. It hence implies that those who get more migrants subsidise the firms who use more native labour. All firms therefore pay equal amount for recruitment of a unit of labour and there is no incentive to delay the entry. This assumption may not depict a completely real situation but it is useful in simplifying analysis following our main goal.

The total cost of a firm producing variety $i$ therefore is,

$$c(i) = WL(i) = F_d W + Wa_y(i)$$

The revenue and operating profits are respectively,

$$r(i) = p(i)y(i) \text{ and } \pi(i) = p(i)y(i) - c(i)$$

Differentiating the operating profit with respect to $p(i)$ and after necessary calculation, price as a function of marginal cost $a$ is obtained as,

$$p(a) = W \frac{a}{\rho}$$

(9)

An interesting feature of the above equation is that the price function contains no information of the varieties. Price is same for any other firm producing a different variety with same marginal cost. Now by using the equation (9), the revenue and operating profits of the firm are obtained as,

$$r(a) = AW^{1-z}\left(\frac{a}{\rho}\right)^{1-z} \text{ and } \pi(a) = BW^{1-z}a^{1-z} - F_d W$$

where, $B = A(1-\rho)\rho^{z-1}$

Thus both the revenue and operating profits of the firms are higher if the firms face a lower average wage rate.
2.4 Firm’s Entry, Exist and Productivity Distribution

Assume that is upon entry the firms in host country draw the marginal cost $a$ from a country specific cumulative distribution $G(a)$. The entrants pay an irreversible fixed cost $F_k$ in term of labour unit, which is identical across all countries. The decision to stay or exit depends on whether operating profit is positive or not. Let $a_B$ be the marginal cost required for the zero operating profit. Thus the 'survival ceiling' $a_B$ is given by,

$$\pi(a_B) = BW^{1-\varepsilon}a_B^{1-\varepsilon} - F_B W = 0 \tag{10}$$

Any marginal cost above survival ceiling $a_B$ implies that a firm makes negative operating profit hence would not carry out the production. Following Falvey et al. (2005, 2006 and 2011) and Melitz and Ottaviano (2008) we parameterise the technology by assuming that $G(a)$ follows a Pareto distribution which common shape parameter $k \geq 1$ but with a cost upper bound $\bar{a}$, that is,

$$G(a) = \left(\frac{a}{a}\right)^k$$

where $\alpha \in [0, \bar{a}]$. As mentioned by Melitz and Ottaviano (2008) the shape parameter $k$ shows the dispersion of cost draw. When $k=1$, the cost distribution is uniform. As $k$ increases, the relative number of high-cost firms increases and the cost distribution is more concentrated on high cost level. Any truncation from above retains the same distribution function and shape parameter $k$. The cost distribution of surviving firms is therefore also Pareto with shape parameter $k$. The truncated cost distribution of surviving firms in equation (10) is therefore,

$$G(a_B) = \left(\frac{a}{a_B}\right)^k \text{ where } \alpha \in [0, a_B]$$

The conditional probability of successful entry is given as $G(a)/G(a_B) = \left(\frac{a_B}{a}\right)^k$.

Above we defined the cost distribution of the host country. For our analysis we also need to look at the cost distribution of the source country. Let us denote the country using ‘Tilda’. Assume that the source country is inferior in technology as such $G(a) > \tilde{G}(a)$. To ensure that we assume that $\bar{a} \leq \tilde{a}$ that is the cost upper bound is higher for source country. Thus the technological gap between the countries is given as,

$$\mu = \frac{G(a)}{\tilde{G}(a)} = \left(\frac{\bar{a}}{\bar{a}}\right)^k = \left(\frac{\tilde{a}}{\bar{a}}\right)^k \geq 1 \tag{11}$$
Unlike Falvey et al. (2005, 2006 and 2011), the measurement of technological gap is not of prime importance to us. We are here looking mainly at the relationship of 'survival ceiling' with the average wage rate and migration.

3. Autarky Equilibrium with Migration and Remittances

In the previous section we have introduced the model and described the basic equations. In this section we look at the impact of migration on the survival ceiling, price level and the welfare under autarky. Autarky is a situation where countries of consideration do not engage in trade. In this paper autarky is defined in slightly different way as we assume that temporary migrants take their earnings back to the source country, though there exists no formal trade between source and host countries. We denote autarky using subscript 'A' implying autarky and migration. In case of open economy the subscript is 'T' implying trade and migration. We do not use any subscript to describe migration as it is considered in both cases. The effects of migration in autarkic situation are analysed in following subsections.

3.1 Equilibrium Demand, Wage Rate and Labour Allocation

Define the autarkic wage rate of host country as \( w_H \), the corresponding wage for the source country is \( \tilde{w}_A \). The migrants in host country receive wage rate \( (w_H - s) \), the total income of \( M \) number of migrants is given as \( (w_H - s)M \). In autarky migrants must use up all income in the host country by purchasing goods from the host country. The total income of the native/domestic labour is \( w_HL \). The total output of the country is equal to the total expenditure of the country. Hence,

\[
H_A + P_A Y_A = w_HL + (w_H - s)M = w_A(L + M) - sM = \Phi_A \tag{12}
\]

where \( \Phi_A \) shows the market size of the host in autarky. The market size with migration is larger than the market size without migration. The solutions of equation (6) is modified as,

\[
Y_A = \frac{P_A}{P_A} \beta \Phi_A \quad \text{and} \quad H_A = (1 - \beta) \Phi_A \tag{13}
\]

As market size is larger there is an expansion of production in homogenous sector with migration. Migrants only work in the heterogeneous sector but they demand goods from both homogenous and differentiated sectors. This increased demand must be supported by increased production in the homogenous sector. As migrants can not work in homogenous sector the additional labour is obtained by drawing domestic labour from differentiated sector. In order to find the equilibrium employment in homogenous sector, we equate the
demand and supply equations of homogenous sector and then using the fact that
\( \bar{L} + M = L_H + L_Y \) obtain,
\[
L_H = (1 - \beta) [\bar{L} + M (1 - a_{hs})]
\]

As \( w_A = \frac{1}{a_H} \), we have \( 0 < 1 - a_{hs} < 1 \). Thus, the equation shows if the supply of migrants increase, the employment in homogenous sector increases and it is positively related to \( M \) and negatively related to \( s \). The employment in differentiated sector is obtained as,
\[
L_Y = \beta \bar{L} + M - M (1 - \beta) (1 - a_{hs})
\]

Hence the heterogeneous sector also experiences an increase of employment. The native labour working in differentiated sector is given by,
\[
\bar{L} - L_H = \bar{L} - (1 - \beta) [\bar{L} + M (1 - a_{hs})] = \beta \bar{L} - M (1 - \beta) (1 - a_{hs})
\]

We can compare it with \( \bar{L} - L_H = \beta \bar{L} \), which is the employment of native workers in the differentiated sector without migration. Thus with migration the differentiated sector experience a decline in native’s participation although as long as \( \beta \bar{L} > M (1 - \beta) (1 - a_{hs}) \) we have native labour working in the differentiated sector. If \( \beta \bar{L} = M (1 - \beta) (1 - a_{hs}) \) all native labours are located in the homogenous sector. We maintain the assumption that \( M \) is sufficiently low so that a number of native labours always work in differentiated sector.

The average wage rate in differentiated sector is thus given as,
\[
W_A = \frac{w_A (\bar{L} - L_H) + (w_A - s) M}{L_Y} = w_A - \frac{s M}{L_Y}
\]

The above equation is the solution of the wage rate with migration as stated previously in equation (8). If migration is zero the average wage is equal to host country’s wage rate. By totally differentiation equation (14) and keeping \( s \) unchanged we obtained,
\[
\frac{dW_A}{dM} = -\frac{s M (1 - (1 - \beta) (1 - a_{hs}))}{L_Y^2} - \frac{s}{L_Y} < 0
\]

Thus if migration increases average wage decreases.

3.2 Cut Off Productivity in Differentiated Sector

Upon entry paying the fixed cost \( F_E \) in term of labour unit, a firm obtains the productivity \( \frac{1}{a} \) from the distribution \( G(a) \). If the firm’s productivity is above or equal to the threshold level \( \frac{1}{a^*_E} \) (equivalently below or equal to survival ceiling \( a^*_D \)), the firm
operate and earn non-negative operating profit. Otherwise the firm immediately leaves and earns zero profit. The entry drives the expected profit net of entry cost to zero. Unlike Melitz (2003) we do not here consider the exogenous shock implying a probability of death of incumbents and proportional entry in each period. The model we use is also not a dynamic model. Thus entry here implies entry as such the expected profit is zero. The term $F_k$ here however requires special attention. As $F_k$ shows the amount of labour used, the total cost of entry is $F_k$ multiplied by the wage rate. Melitz modelled it as the initial sunk investment which firms must make before participating in the draw to obtain productivity. The cost of initial investment therefore is expected to be made using the native labour force not the migrants. Zero expected profit gives,

$$E(\pi_A) = \int_0^a \pi(a) dG(a) + \int_{a^b}^a dG(a) = w_A F_k$$  \hspace{1cm} (16)

A firm operates only if the marginal cost is below or equal to $a^b$. When marginal cost is $a^b$ the firm earns zero operating profit. This is defined as zero cut off profit condition. Utilising the zero cut of profit condition and after necessary calculations we have,

$$E(\pi_A) = F_k W_A Q(a^b)$$  \hspace{1cm} (17)

where $Q(a^b) = \int_0^{a^b} \left[ \left( \frac{a^b}{a} \right)^{K-1} - 1 \right] dG(a)$

Integrating by parts we get,

$$Q(a^b) = \left[ \frac{a^b}{a} \right]^K (K-1)$$  \hspace{1cm} (18)

Substituting equation (18) in the free entry condition we obtain,

$$\left[ \frac{a^b}{a} \right]^K = \frac{w_A F_k}{F_k W_A (K-1)}$$

or,

$$a^b = a \left( \frac{w_A F_k}{F_k (K-1)} \right)^{\frac{1}{K}} W_A ^{-\frac{1}{K}}$$  \hspace{1cm} (19)

The equation (19) shows that the long run survival cut off productivity of firm is higher compare to that in no migration situation. The survival ceiling however does not depend on the market size. To see how marginal changes in migration alter the equilibrium survival productivity we obtain through total differentiation,

$$\frac{da^b}{dW_A} = - \frac{1}{K} \left( \frac{wa_F_k}{F_k (K-1)} \right)^{\frac{1}{K}} W_A ^{-\frac{1}{K}K < 0}$$

Thus the survival productivity is inversely related to the average wage rate. If average
wage is higher survival productivity is lower and vice versa. As average wage is inversely related to the number of migrants, an increase in the number of migrants means an increase in the survival ceiling. Thus with migration, the more firms can survive compared to no migration situation which is expressed in the following proposition:

**Proposition 1:** As average wage rate in differentiated sector decreases because of increases intake of migrants, the survival ceiling of the host country increases.

### 3.3 Number of Firms

Let the number of entrant denoted by $N^4_k$. Each firm produces a new variety and get the productivity from the distribution $G(a)$. The price for the firm with marginal cost $a$ is $p(a) = \frac{W_a a}{\rho}$. The aggregate price in term of $N^4_k$ entrants can be written as,

$$P = \left[ \int_{i \in \mathcal{V}} p(i)^{1-z} \, di \right]^{\frac{1}{1-z}} = \left[ N^4_k \int_0^{a^4_k} \left( \frac{W_a a}{\rho} \right)^{1-z} \, dG(a) \right]^{\frac{1}{1-z}}$$

Let us write $v(a^4_k) = \int_0^{a^4_k} a^{1-z} dG(a)$. Then from the equation (20) we obtain

$$v(a^4_k) = \frac{P^4_k}{N^4_k W^4_k \rho^{1-z}}.$$ 
From the zero cut off profit condition in equation (10) and after necessary manipulation we obtain $P^4_k = \frac{\beta \Phi_a (1-\rho) W^4_k a^{4-1-z}}{F \rho^{1-z}}$. Using this result we obtain,

$$v(a^4_k) = \frac{\beta \Phi_a (1-\rho) a^{4-1-z}}{N^4_k W^4_k F_k}$$

On the other hand calculations reveal that,

$$v(a^4_k) = \left[ \frac{a^4_k}{a^4_k} \right]^k \frac{K}{a^4_k}$$

where $K = \frac{k}{k - \varepsilon + 1}$

Therefore by equating the equation (20), (22) and using the solution of survival productivity in equation (19) we get,

$$N^4_k = \left( \frac{a^4_k}{a^4_k} \right)^k \frac{\beta \Phi_a (1-\rho)}{K W^4_k F_k} = \frac{\beta \Phi_a (1-\rho)(K-1)}{w^4_k F_k K} = \frac{\rho \beta \Phi_a}{w^4_k F_k K}$$

Looking at the equation we can see that the number of entrants is higher as the market size is higher with migration. Again we assume that the economy is in equilibrium with migration and analyse how it changes as the number of migrants changes. Therefore by
differentiating totally we get,
\[ \frac{dN_A}{dM} = \frac{\rho \beta}{w_A F_k K} (w_A - s) > 0 \]

Hence number of entrants increases as migration takes place. The long run number of firms is defined by the number of entrants multiplied by the probability of successful entry that is,
\[ N^*_A = \left( \frac{a^*_A}{a} \right)^h N^*_k = \frac{1}{N^*_k} \beta \Phi_A (1 - \rho) \frac{KW_A F_D}{F_d W_A K} N^*_k = \frac{\beta \Phi_A (1 - \rho)}{F_d W_A K} \]

Again by differentiating totally we get,
\[ \frac{dN_A}{dM} = \frac{\beta \Phi_A (1 - \rho)}{F_d W_A K} \left( \frac{\beta (1 - \rho) (w_A - s)}{F_d W_A K} > 0 \right) \]

Thus as number of migrants increase the long run number of surviving firms also increases. In proposition:

**Proposition 2:** As migration increases both the number of entrants and surviving firms of the host country increases.

### 3.4 Aggregate Price of Differentiated Goods

In the calculations of previous sections, the aggregate price has been obtained as,
\[ P_A = \frac{\beta \Phi_A W_A^{-\xi} (1 - \rho)}{F_D \rho^{1-\xi}} \]

which can be written as,
\[ P_A = a^*_A (\beta \Phi_A)^{-\frac{1}{\xi+\kappa}} F_D^{-\frac{1}{\xi}} W_A^{\frac{1}{\xi}} (1 - \rho)^{-\frac{1}{\xi+\kappa}} \rho^{-1} \]

Therefore aggregate price depends on wage, market size and survival ceiling. The price increases as survival ceiling and market size increases and decreases as average wage decreases. By substituting the value of survival ceiling from equation \( \xi \) we get,
\[ P_A = a \left( \frac{w_A F_E}{F_D (K - 1)} \right)^{\frac{1}{\xi}} W_A^{\frac{1}{\xi} + \kappa} (\beta \Phi_A)^{-\frac{1}{\xi+\kappa}} F_D^{\frac{1}{\xi}} (1 - \rho)^{-\frac{1}{\xi+\kappa}} \rho^{-1} \]

By totally differentiating equation \( \xi \),
\[ \frac{dP_A}{dM} = \Psi \frac{1 - s + \kappa s}{K (\xi - s)} (\beta \Phi_A)^{-\frac{1}{\xi+\kappa}} W_A^{\frac{1}{\xi+\kappa} - 1} dW_A - \Psi \frac{1}{\xi - 1} W_A^{\frac{1}{\xi+\kappa} - 1} (\beta \Phi_A)^{-\frac{1}{\xi+\kappa}} (w_A - s) < 0 \]

where
\[ \Psi = a \left( \frac{w_A F_E}{F_D (K - 1)} \right)^{\frac{1}{\xi}} F_D^{\frac{1}{\xi}} (1 - \rho)^{-\frac{1}{\xi+\kappa}} \rho^{-1} \]

(470)
Thus as migration increases price level falls.

3.5 Welfare of the Host Country

Φ_A shows that market size of the whole economy but it contains also the migrants. However from the policy maker’s perspective it is important to know how the welfare of domestic labour changes as migration takes place. The total income of domestic labour is \( w_A L \). Let us denote the consumption of homogenous and differentiated goods by domestic labour as \( H_A' \) and \( Y_A' \). Therefore the demand of the two goods is obtained as,

\[
H_A' = (1 - \beta)w_A L \quad \text{and} \quad Y_A' = \frac{\beta w_A L}{P}
\]

By substituting the demand, as obtained above, in utility function we obtain,

\[
U = H_A'^{1-\beta}Y_A'^{\beta} = [\beta(1-\beta)]^{1-\beta}w_A L P^{-\beta}
\]

Therefore the per capita welfare of domestic labour is,

\[
\frac{U}{L} = [\beta(1-\beta)]^{1-\beta}w_A P_A^{-\beta}
\]

Therefore the welfare of the domestic labour increases when the aggregate price of differentiated goods decreases. As price decreases when migration increases, the welfare of domestic labour increases with migration.

**Proposition 3:** As migration increases price of differentiated goods falls. The welfare of the domestic labour increases with migration.

3.6 Welfare of the Source Country

In this section we look at the benefit of source country. In autarky, the countries cannot trade with each other. But we assume that migrants can return with the goods after purchasing them from the host country. The production and demand in the source country is therefore obtained only in accordance to the labour force and wage rate available in source country.

We assume that both host and source countries exhibit same types of taste, thus the same Cobb-Douglas utility function is applicable. Similar to host, there are two sectors of production in source, one is homogenous sector and the other one is differentiated sector. The homogenous sector exhibits constant returns to scale which gives wage rate \( \tilde{w}_A \). But the demand of differentiated good is of CES love of variety form. The production in differentiated sector is defined by increasing returns to scale similar to host country. Productivities of the firms are now obtained from the distribution \( \tilde{G}(a) \), as has been
discussed in section 2.

We can carry out the whole calculation based on the above assumptions but it is not essential as the results are similar to what we had in the case of host country. It is however essential that we look at the price level to find the effect of migration on welfare of those who is living behind. It is obtained as,

$$P = \tilde{a}^\beta(\beta(\tilde{L} - M))^{-\frac{1}{1-\rho}}F_\mu^{\frac{1}{1-\rho}}w_\mu^{\frac{1}{1-\rho}}(1-\rho)^{-\frac{1}{1-\rho}}\rho^{-1}$$

As the survival ceiling remains unchanged, the price level increases as the market size falls. It is needless to say that the equilibrium number of firms is lower compared to no migration case. As the price level increases the welfare of the people living behind decreases. We can here relate our finding to the ‘brain drain’ literature by stating that with migration of a fraction of population, the welfare of those who left behind decreases. This result is different from the influential paper of Grubel and Scott (1966) that based on a perfectly competitive model stated that the welfare of those who stay in source country remains unchanged.

We may want to see if the total welfare of the nationals of the source country increases or decreases because of migration. By adding the utilities of the domestic labour and the migrants we obtain.

$$U_R + U_M = [\beta^\alpha(1-\beta)]^{-\beta}\tilde{w}_A(\tilde{L} - M)\tilde{P}_A^{-\beta} + [\beta^\alpha(1-\beta)]^{-\beta}(w_A - s)MP_A^{-\beta}$$

$$=[\beta^\alpha(1-\beta)]^{-\beta}(\tilde{w}_A\tilde{L}\tilde{P}_A^{-\beta} - \tilde{w}_A\tilde{M}\tilde{P}_A^{-\beta} + (w_A - s)MP_A^{-\beta})$$

As $\tilde{w}_AM < (w_A - s)M$, if $\tilde{P}_A$ is greater than or equal to $P_A$ the welfare of the source country is higher with migration. In order to have migration taking place we need that the purchasing power of the wage received in host is higher than the purchasing power of the wage receive in source. It implies that $\tilde{P}_A$ is higher than $P_A$. Thus we can express the results in proposition format.

**Proposition 4:** The productivity threshold of the source country remains same but price level increases as market size falls because of emigration. Without remittances the welfare of people who left behind falls. If migrants return back with their income the overall welfare of the nationals of source increases if the differentiated goods are relatively cheaper in the host country.

4. Open Economy with Migration and Remittances

In the previous section we have described a special autarkic situation where the
countries can send migrants and receive remittances in term of good but cannot trade with each other. In this section we modify this assumption such that host and source countries not only exchange migrants but also can trade with each other. In trade theory, the transition from autarky to trade is done to show the changes in factor allocation, production and the gain from trade. Our aim differs from that as we look at the changes in economy that migration brings. That is if migration takes place how it changes the economy wide variables when the countries trade with each other. Here we only consider the trade and migration between two countries. But with some manipulation the model may be extended to include countries that only engage in intra-industry trade without any migration.

4.1 Basic Assumptions, Market Size and Equilibrium Wage Rate

As before we denote the source country using Tilda and the host country without Tilda. We use subscript 'T' to imply trade. For example, \( w_T \) denotes the wage rate of host country and \( \tilde{w}_T \) denotes the wage rate of source country with trade.

We assume that both countries engage in trade of both goods. We assume standard iceberg transport trade cost \( t > 1 \) for differentiated sector implying, for example, to export \( X \) amount of good, \( tX \) amount of goods must be shipped. On the other hand there is no trade cost associated with homogenous good, which is a standard simplifying assumption. It is also essential that both countries are large enough and the demand is such that they both produce homogenous goods from which the wage rates of domestic workers can be derived.

As there is no trade cost, price of homogenous good is same in both countries. As in the case of autarky, the price of homogenous good is selected as numeraire. This gives the wage rates \( w_T \) and \( \tilde{w}_T \) which are different because of technological differences in homogenous sector. For migration we require \( w_T > \tilde{w}_T \). As before migrants receive lower wage compared to the natives defined as \( (w_T - s) \). However as income of migrants is higher compared to the source country’s wage rate, it must be followed by increased demand of homogenous and differentiated goods. In the case of autarky it had initiated internal migration of native labour force from heterogeneous to homogenous sector. It may not be the case in open economy. We assume that the sizes of the economies and the amount of migration are such that the additional demand is entirely supported by production in the migrants’ source country. Thus the numbers of native labour working in the homogenous and differentiated sector of host country remain the same. Through these assumptions, it is possible for us to determine the average wage rate of differentiated goods sector in a simplified manner without figuring out the trade equilibrium between two countries. Hence, the average wage of heterogeneous sector of host country is given by
here is the number of native labour of host country working in differentiated sector.

We consider one period of migration and assume that at the same period the migrants return back to country after working and receiving wage. The wage received in host country can be considered as a 'token' that allows them to purchase goods from both countries. The market size of the host is then given by,

\[ H_T + P_T Y_T = w_T L = \Phi_T \]

Similarly the market size of source is given by,

\[ \tilde{H}_T + \tilde{P}_T \tilde{Y}_T = \tilde{w}_T (L - M) + (w_T - s) M = \tilde{\Phi}_T \]

The above assumption regarding the market size is not completely at par with common observation that migrants do spend some income in the host country. But it serves one important purpose. Remittances without return migration should increase the per capita welfare of the remaining migrants in the origin. The assumption of return migration with complete repatriation of remittances allows us to isolate this effect.

4.2 Zero Cut Off Profit Conditions

We now look at the zero cut off profit conditions in source and host countries. It is standard in the literature to assume that in order to operate in foreign country the firm must bear a fixed cost which is independent of export volume. Let us denote it as \( F_X \) in units of labour. Melitz (2003) and Helpman et al. (2004) referred it as the cost incurred in forming distribution and service networks in the foreign country. Similar costs in the home country are included in \( F_B \). The total fixed cost for operating in the foreign market is given as \( F_X \) times wage rate, hence we need to decide which wage rate is to be multiplied to \( F_X \). In the usual models the wage rate is assumed as 1 thus this matter has not received attention. But in our model wage rate differs. As distribution and forming networks in the foreign market can use foreign labour force, the wage rate in production and the wage rate in forming networks and distribution can differ. For example in the host country, the wage rate in production is \( W_T \), but for distribution in the foreign market it may only pay \( \tilde{w}_T \). To simplify calculation we throughout assume same wage rates in production and in foreign operations keeping in mind that they can differ resulting in substantial changes in calculations.

The zero cut off profit condition of equation (10) can be redefined for the host as,

\[ a^*_D \tau^{1-z} W_T^{1-z} B_T = W_T F_B \]

(27)
For the home country the corresponding equation is,

$$\tilde{a}_b^{\tilde{r}-\tilde{w}} w_T^{1-\tilde{w}} \tilde{B}_T = \tilde{w}_T F_d$$

(28)

where $B_T = A_T (1-\rho)^{\phi^{-1}}$, $\tilde{B}_T = \tilde{A}_T (1-\rho)^{\phi^{-1}}$, $A_T = \beta w_T \tilde{L}_T \tilde{P}_T^{\phi^{-1}}$ and $\tilde{A}_T = \beta \tilde{\phi}_T \tilde{P}_T^{\phi^{-1}}$. With iceberg transport cost $t>1$ and a fixed cost $F_X$ in unit of labour for export we have,

$$\pi_X(a) = (at)^{1-\epsilon} W_T^{1-\epsilon} \tilde{B}_T - F_X \tilde{w}_T$$

The equation implies that the firms of host country take the market size of the source into consideration when exporting. Similarly for the export from the source to the host country we get,

$$\tilde{\pi}_X(a) = (at)^{1-\epsilon} \tilde{w}_T^{1-\epsilon} \tilde{B}_T - F_X \tilde{w}_T$$

The above two equations generate export survival ceilings $a_X$ and $\tilde{a}_X$ where operating profits are zero. From them we get,

$$(a_X)^{1-\epsilon} W_T^{1-\epsilon} \tilde{B}_T = F_X t^{1-\epsilon} = (\tilde{a}_X)^{1-\epsilon} \tilde{w}_T^{1-\epsilon} \tilde{B}_T$$

(29)

Utilising equations (27), (28) and (29) we get,

$$\frac{a_b}{a_x} = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\tilde{r}}^{\frac{1}{\tilde{r}-\epsilon}} = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\epsilon} \phi = v_1$$

(30)

and,

$$\frac{\tilde{a}_b}{\tilde{a}_x} = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\tilde{r}}^{\frac{1}{\tilde{r}-\epsilon}} = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\epsilon} \phi = v_2$$

(31)

where $\omega = \frac{W_T}{\tilde{w}_T}$, $f = \frac{F_X}{F_d}$, $\phi = t f_{\epsilon} \tilde{r}$, $v_1 = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\epsilon} \phi$, $v_2 = \omega^{\frac{\tilde{e}}{\epsilon}} t f_{\epsilon} \phi$

Thus $\omega$ stands for relative wage and $f$ stands for the ratio of export cost to the domestic operating cost. In the symmetric case where market size and wage rates are the same, we get $a_b = a_x$ and $a_X = \tilde{a}_X$. If $\phi>1$ then in symmetric case it must be the case that $a_b > a_X$ and $\tilde{a}_b > \tilde{a}_X$ implying the export requires higher threshold productivity level.

With asymmetry in market size and wage rates, comparison of the productivities of domestic firms and exporters are not so straight forward. But we can compare productivity levels of domestic and foreign firms within an economy. As $\omega>1$ and $\phi>1$ we have $a_b > a_X$ and $\tilde{a}_b > \tilde{a}_X$. That is firms exporting in an economy are more productive compared to the domestic firms operating in that economy. Migration lowers the ratio $\omega$ which changes the productivity gap between the foreign and domestic firms.
4.3 Equilibrium Productivity With Migration

Firms enter in domestic and foreign markets as long as the expected operating profit of the firms net entry cost is equal to zero. In the long run we have,

\[ E(\pi_T) = \int_0^{\Delta \pi_T} \pi_D(a) dG(a) + \int_0^{\Delta \pi_X} \pi_X(a) dG(a) = F_T W_T Q(a) + F_X W_T Q(ax) = \pi_T F \]

For the source country the equivalent equation is,

\[ E(\pi_T) = \int_0^{\Delta \pi_T} \pi_D(a) dG(a) + \int_0^{\Delta \pi_X} \pi_X(a) dG(a) = F_T \pi_T Q(a) + F_X \pi_T Q(ax) = \pi_T F \]

The first terms of above two expressions are expected domestic profit and the second terms are expected export profit. In the similar manner as in equations (17) to (19) and using equations (20) and (21) we get,

\[ W_T \left[ \frac{a_T^a}{a} \right]^k + W_T \omega^{\frac{\kappa s}{\mu T \mu}} \left[ \frac{\pi_T}{a} \right]^k = \frac{w_T F_E}{F_D(K-1)} \]

where \( \Omega = \phi^{-k} f = t^{-k} f^{\frac{k+z-1}{k-z}} \). It can be shown that \( 0 \leq \Omega \leq 1 \) as \( t \geq 1 \) and \( F_X > F_D \). Similarly by carrying out the calculation for the source country we obtain,

\[ \left[ \frac{\pi_T}{a} \right]^k + \omega^{\frac{\kappa s}{1-z \mu}} \Omega \left[ \frac{a_T^a}{a} \right]^k = \frac{F_E}{F_D(K-1)} \]

From equation (34) and (35) the survival ceiling for domestic firms of the host is obtained as,

\[ \left[ \frac{a_T^a}{a} \right]^k = \Lambda_1 \left( \frac{w_T}{W_T} \right)^{\frac{\kappa s}{1-z \mu}} \Omega \]

where \( \Lambda_1 = \frac{W_A}{(1-\Omega^2)w_A} \left[ \frac{a_T^a}{a} \right]^k \). Here \( \left[ \frac{a_T^a}{a} \right]^k \) is the survival ceiling in case of autarky. As \( \Omega \leq 1, \Lambda_1 \) is positive. The figure in the bracket must be positive to have positive survival ceiling. When technological gap is nil that is \( \mu = \left( \frac{a}{a_T} \right)^k = 1 \), the second part of the figure in bracket is smaller than unity as \( W_T > \pi_T \). So positive sign is automatically ensured. When \( \mu \geq 1 \) that is technological gap exists, we need the assumption that \( \mu \) is not too large relative to \( \Omega \) and the wage ratio. Now we are interested to know how migration changes the above survival ceiling. By differentiating equation (36) we get,
The equation shows how migration alters the survival ceiling. The sign of the expression depends on \( \frac{w_T}{W_T^2} - \frac{k\varepsilon}{\varepsilon-1} \) as other parts are positive by assumption. Thus migration increases the survival ceiling of domestic firms if \( \frac{w_T}{W_T^2} > \frac{k\varepsilon}{\varepsilon-1} \). The survival ceiling for the source country using equations \( c2oe3 \) and \( c2oe4 \) is obtained as,

\[
\left[ \frac{\tilde{a}_T}{\widetilde{a}} \right] = \Lambda_2 \left( 1 - \frac{w_T}{W_T} \left( \frac{W_T}{w_T} \right)^{\frac{h_k}{\varepsilon-1}} \mu^{-1} \right) \tag{33}
\]

where \( \Lambda_2 = \frac{1}{1-\Omega^2} \left[ \frac{\tilde{a}_T}{\widetilde{a}} \right] \).

Again we assume that the term in bracket is positive to ensure positive survival ceiling. In the numerator \( 0<\mu^{-1} \leq 1 \) and \( 0<\Omega \leq 1 \), hence if wage ratios are low enough then term in bracket is positive. By totally differentiating we get,

\[
\frac{\tilde{a}_T}{dM} = \frac{1}{r} a \Lambda_2 \frac{sL_{VH}}{(L_{VH}+M)^r} \left( 1 - w_T w_T \frac{h_k}{\varepsilon-1} W_T^{1-\varepsilon+\frac{h_k}{\varepsilon-1}} \mu^{-1} \Omega \right)^{1-h} \left( 1 - \frac{1-\varepsilon+k\varepsilon}{\varepsilon-1} + W_T^{\frac{2-2\varepsilon+k\varepsilon}{\varepsilon-1}} \right) \tag{34}
\]

By assumption \( (1-w_T w_T) W_T^{\frac{2-2\varepsilon+k\varepsilon}{\varepsilon-1}} \mu^{-1} \Omega \) is positive. We have \( \frac{1-\varepsilon+k\varepsilon}{\varepsilon-1} < 0 \). Thus as migration increases the survival ceiling of domestic firms of the source country falls. Using equations \( c2oe0 \) and \( c2oe9 \) we get,

\[
a_x = \frac{\tilde{a}_T}{v_2} = \omega \phi^{-1} a \Lambda_2 \left( 1 - \Gamma_1 \right)^{\frac{1}{h}} \tag{35}
\]

where, \( \Gamma_1 = w_T w_T \frac{h_k}{\varepsilon-1} W_T^{1-\varepsilon+\frac{h_k}{\varepsilon-1}} \mu^{-1} \Omega \). By totally differentiating we get,

\[
\frac{dax}{dM} = \phi^{-1} a \Lambda_2 \frac{sL_{VH}}{(L_{VH}+M)^r} \left( \frac{1-\varepsilon+k\varepsilon}{\varepsilon-1} \right)^{\frac{1}{h}} \left( 1 - \frac{1-\varepsilon+k\varepsilon}{\varepsilon-1} + W_T^{\frac{2-2\varepsilon+k\varepsilon}{\varepsilon-1}} \mu^{-1} \Omega \right) \left( 1 - \Gamma_1 \right)^{\frac{1}{h}} \frac{\varepsilon}{\varepsilon-1} w_T^{\frac{2-2\varepsilon+k\varepsilon}{\varepsilon-1}} \right) \tag{477}
\]

The expression is long but as we can see easily the sign is positive implying that the
survival ceiling of exporting firms of host increases because of migration. For the exporting firms of source country we get,

$$a_x = \omega^{\frac{1}{z-\varepsilon}}\phi^{-1}a \Lambda \left( \frac{w_T}{W_T} - \left( \frac{W_T}{w_T} \right)^{-\frac{\mu}{\varepsilon-1}} \mu \Omega \right)^{\frac{1}{z-\varepsilon}}$$  \(\text{(39)}\)

By differentiating we get,

$$\frac{d\tilde{a}_x}{dM} = \phi^{-1}a \Lambda \left[ \frac{\varepsilon}{z-\varepsilon} \left( \frac{w_T}{W_T} - \left( \frac{W_T}{w_T} \right)^{-\frac{\mu}{\varepsilon-1}} \mu \Omega \right)^{\frac{1}{z-\varepsilon}} \right] \frac{sL_{\text{ym}}}{(L_{\text{ym}}+M)^{\varepsilon-1}}$$

where $\Gamma = \left( \frac{w_T}{W_T} - \left( \frac{W_T}{w_T} \right)^{-\frac{\mu}{\varepsilon-1}} \mu \Omega \right)^{\frac{1}{z-\varepsilon}}$. Thus if survival ceiling of domestic firms in host falls the survival ceiling of exporting firms in host country also falls. Other wise survival ceiling may increase. We can summarise the results in following proposition:

**Proposition 5:** As migration increases the export survival ceiling of the host country and domestic survival ceiling of the source country increase. The export survival ceiling of the source falls if the domestic survival ceiling of the host falls.

The proposition summarises the results. But we need further discussion in this regard. A well established result in the literature is that domestic survival ceiling of firms falls in transition from autarky to open economy. An intuition of the situation of this paper can be obtained if we compare equations (36) and (32). In both cases the firms enter until the expected profit is equal to the entry cost. In autarky the firms operate only in domestic market while in open economy the firms operate both in domestic and export market. Because of the division of expected profit between exporter and non-exporters, the non-exporters, that is, domestic firms now survive if marginal cost is low. Similar thing happens with migration. Migration increases the export survival ceiling of host firms. Without migration the survival ceiling of the domestic firms falls. But as migration lowers the operating cost it is also possible for domestic firms to enjoy higher survival ceiling due to migration.

With migration the survival ceiling of domestic firms of the source country falls. Firms in an open economy exports and operate in domestic market in a way that expected profit is zero. Therefore the fall in survival ceiling of domestic firms implies that domestic firms are more productive. Survival ceiling of exporters of the source country should increase implying that less productive firms of the source survive in export market. However this relationship is not clearly observed with migration. With migration survival ceiling may also fall implying that only more productive firms can survive with migration.
4.4 Number Active Firms

Let us assume that the mass of entrants in host and source is given by \( N^T_E \) and \( \tilde{N}^T_E \). The aggregate price in term of number entrants can be written as,

\[
P^{1-z} = \int_{i=0}^{a^E} p(i)^{1-z} di = \int_{0}^{a^E} N^T_E \left( \frac{W_T a}{\rho} \right)^{1-z} d(G(a)) + \int_{0}^{a^E} \tilde{N}^T_E \left( \frac{\tilde{W}_T a}{\rho} \right)^{1-z} d(\tilde{G}(a))
\]

\[
= N^T E W^T T^{1-z} v(a^T) \rho^{z-1} + \tilde{N}^T E \tilde{W}^T T^{1-z} \tilde{v}(\tilde{a}_x) \rho^{z-1} t^{1-z}
\]

where \( v(a^T) = \int_{0}^{a^E} a^{1-z} d(G(a)) \) and \( \tilde{v}(\tilde{a}_x) = \int_{0}^{\tilde{a}_x} \tilde{a}^{1-z} d(\tilde{G}(a)) \). As \( a^T_{1-z} W^T T^{-z} B_T = F_B \), by substitution and after necessary calculation we obtain,

\[
\left[ \frac{a^T}{a} \right]^k = \frac{\beta w_T L(1-\rho)}{F_B K [N^T_E W^T T + \tilde{N}^T_E \tilde{W}^T T^{1-\omega} - \frac{k}{2} \Omega]}
\]

and,

\[
\left[ \frac{\tilde{a}_x}{\tilde{a}_x^T} \right]^k = \frac{\beta \tilde{v}(1-\rho)}{F_B K [N^T_E W^T T^{1-\omega} + \tilde{N}^T E \tilde{W}^T T^{1-\omega} \Omega]}
\]

By carrying out the calculation further we get following two equations,

\[
N^T E W^T T + \frac{W_A}{w_A} \delta^{-1} N^T E \tilde{W}^T T^{1-\omega} - \frac{k}{2} \Omega = \frac{\beta w_T L(1-\rho)}{F_B K}
\]

and, \( \delta \frac{W_A}{w_A} N^T E W^T T^{1-\omega} \Omega + \tilde{N}^T E \tilde{W}^T T^{1-\omega} \Omega = \frac{\beta \tilde{v}(1-\rho)}{F_B K} \)

where \( N^T = \left[ \frac{a^T}{a} \right]^k \tilde{N}^T = \left[ \frac{\tilde{a}_x}{\tilde{a}_x} \right]^k \tilde{N}^T \) and \( \delta = \frac{1 - \frac{w_T}{w_T} \frac{w_T}{w_T} - \frac{k}{2} \mu^{1-\Omega} \Omega}{\frac{w_T}{w_T} - \frac{k}{2} \mu^{1-\Omega} \Omega} \). Thus \( N^T \) and \( \tilde{N}^T \) are number of surviving firms in host and home countries. By solving we obtain,

\[
N^T = \frac{1}{w_T (1-\Omega^2)} \frac{\beta w_T L(1-\rho)}{F_B K} \left( 1 - \left( \frac{\tilde{v}}{w_T L} \right) \frac{w_A}{W_A} \frac{\delta^{-1} \mu^{-1} \Omega}{w_T} \right) \frac{k}{2}
\]

and

\[
\tilde{N}^T = \frac{1}{\tilde{w}_T (1-\Omega^2)} \frac{\beta \tilde{v}(1-\rho)}{F_B K} \left( 1 - \left( \frac{\tilde{v}}{w_T L} \right) \frac{W_A}{w_A} \frac{\delta \mu \Omega}{w_T} \right) \frac{k}{2}
\]

Equations (40) and (41) can be differentiated to analyse the effects of migration. But the calculation is very cumbersome to present and simple visual inspection reveals some important information. The sign of the both expressions must be positive to have positive number of firms. In equation (40) the market size of source has negative effect on the
number of firms in host. On the other hand if average wage in the host falls, then number of firms in the host increases. In equation (40) the opposite happens as market size of the source has positive effect and wage ratio has negative effect on firm’s number. The overall change also depends on \( \delta \). Differentiation reveals (not shown) that \( \delta \) falls with migration. Thereby overall number of surviving firms depends together on \( \Phi \), \( \omega \) and \( \delta \).

4.5 The Price Level and Welfare of the Host Country

The expression for price level is similar to the expression we obtained in the autarky that is,

\[
P_T = a^T (\beta w_T L)^{-\frac{1}{1+\tau}} F_B^{\frac{1}{1+\tau}} W_T^{\frac{\tau}{1+\tau}} (1-\rho)^{-\frac{1}{1+\tau}} \rho^{-1}
\]

Using equation (42) we obtain,

\[
P_T = a^T \bar{\Lambda}^T \left( \frac{w_T}{W_T} \right)^{\frac{\mu_\Omega}{1+\tau}} \left( \beta w_T L \right)^{-\frac{1}{1+\tau}} F_B^{\frac{1}{1+\tau}} W_T^{\frac{\tau}{1+\tau}} (1-\rho)^{-\frac{1}{1+\tau}} \rho^{-1}
\]

In order to evaluate the effect of migration we may totally differentiate equation (43) and evaluate the total change in price level given the total change in migration. The expression is again long but simple inspection of equations (42) and (43) can give us indication about the possible effect of migration. We already know that wage rate falls with migration. Hence it should effect price level negatively. However \( a^T \) may increase or decrease with migration. If \( a^T \) falls then price level definitely falls. The intuition behind the proposition is simple. If survival ceiling falls, only the more productive firms can operate in market. Thereby price level falls. In addition decreasing wage rate influences price level negatively. But if survival ceiling increases less productive firms manage to stay in the market. So price level may increase if the effect of increase in survival ceiling completely counteracts the effect of the fall of wage rate. It follows therefore if price level falls, the welfare of the host increases and vice versa.

4.6 Welfare of the Source Country

The price level of the source country is given by,

\[
\tilde{P}_T = a^T w_T^{\frac{\tau}{1+\tau}} (\beta \tilde{\Phi}_T)^{-\frac{1}{1+\tau}} (1-\rho)^{-\frac{1}{1+\tau}} \rho^{-1} F_B^{\frac{1}{1+\tau}}
\]

Again by inspection we can see how migration affects the price level. In one hand migration increases the survival productivity implying increase in the price level. However migration implies lower price level via the increased market size. Thus overall effect of migration on price level is ambiguous. The utility of source is given as,
\[ U_T = \beta^\alpha (1 - \beta)^{1-\beta} (w_T (L - M) + (w_T - s) M) L^{-\beta} \]

Thus utility is higher if price is lower and migration is higher. But we can not unambiguously determine the changes in welfare. In proposition,

**Proposition 6:** Emigration increases the survival ceiling of domestic firms of the source country which raises the price level. Increase in migration on the other hand decreases price level via increased market size. Thus overall effect of migration on price level and welfare of source country is ambiguous.

5. Conclusion

The paper analysed the effect of migration between two countries within the framework of a monopolistic competition model with heterogeneity of productivity across countries and firms. The paper provided us with a method of analysing the economic benefit of movement of people from one country to another country which is feasible within the framework of bilateral or multilateral migration agreements.

International migration is a highly contentious issue and agreements usually are lacking. A few studies have, using simulation, tried to evaluate the economic benefit of international migration on global and regional scales. But there is a shortage of works that can provide guidelines to individual countries. Our study aims towards that gap.

Our analysis showed that in autarky that is when the countries do not trade with each other then migration from low wage source (technologically backward) to high wage host (technologically advanced) country increases the welfare of the host country. Migration also allows the relatively inefficient firms to survive by lowering the threshold productivity level. On the other hand the productivity threshold remains the same in the source country and the people living behind experiences a reduction of welfare as price level increases due to the fall or market size. However when the migrants return back to source with remittances, the total welfare that is the sum of welfare of migrants and non-migrants of source may go up. This result is opposite of what obtained in the influential paper of Grubel and Scott (1966) that argued that the welfare of the people staying behind remain unchanged. In open economy that is when the countries trade with each other the result is not so straight forward. We obtained that migration lowers the productivity threshold of exporter of the host. However migration may or may not lower the productivity threshold of the domestic firms of the host. On the other hand migration lowers the productivity threshold of the domestic firms of source but may or may not decrease the same for the exporters of source country. The effects of migration on the
welfare and price levels of the source and host were ambiguous.

The study has therefore provided us with some understanding of how migration may affect the economies of sending and receiving countries. It is hence adding up to the ongoing debate on the issue surrounding international migration. The study is however looking at effects of international migration between two nations. But in reality migration can also take place between the countries that do not trade much with each other. Migration of labour to a country may also affect the international trade of countries where migration do not take place. In this sense the study is a bit limited as it is incapable of capturing these scenarios. To obtain a stronger grasp of the effect of international migration, models needs to be developed that will enable analysis in a multi-country setting. An extension of the model in that direction will be highly useful in migration related dialogues.

Note
2) For guidance on calculation and interpretation see Brakman and Heijdra (2004).

Reference


