THE SPATIAL STRUCTURE, CITY SIZE AND CITY NUMBER IN A HIERARCHICAL INTER-URBAN SYSTEM

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Abstract

This paper presents a simple model of a hierarchical inter-urban system that is tree-shaped, consisting of a few ranks of cities constituted by differentiated business firms and households. By supposing that the firm at any city rank needs to transport its intermediary inputs from the firms at the next higher rank, and to communicate with the households at the same rank, an equilibrium of the spatial structure is obtained, with the city size and city number depending on the transportation and communication costs. It is also shown that throughout all city ranks of the urban system, by the market principles, the city size would be too large while the city number would be too small when compared to the social optimum.

Key Words: urban hierarchy, spatial structure, city size, city number, market equilibrium, social optimum

1. INTRODUCTION

To study the regional and urban economies of any country, one should take notice of the importance of its urban hierarchy. Generally, such a hierarchical system consists of a number of cities, which are spatially distributed, with different city sizes and city numbers at different city ranks. So, to get a whole understanding of the urban hierarchy, we should grasp the mechanism that governs the spatial structure, city size and city number within the system. This, however, appears to be a very difficult task, and in the first place, we need to review and absorb the essence of related research work that have been carried out so far.

Concerning the existing studies on the urban system, according to Mulligan (1984), the related literature can be classified into three levels: firm-level, settlement-level and system-level. In the firm-level analysis, the formation of urban hierarchy is explained from the view of maximizing behavior of economic actors. Typical examples of firm-level analyses are from the work of Christaller (1933), Lösch (1954), Eaton and Lipsey (1982), and recently Fujita

I wish to dedicate this paper to Professor Noboru Sakashita, who passed away recently in August 2003, for his valuable comments on an earlier version of this paper and his numerous advices on my research work so far. I alone, however, am responsible for any remaining errors.

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(1996). In contrast, the settlement-level analysis focuses on the economic characteristics of settlements at different levels of the given central place system. For instance, Alperovich (1982), Mulligan (1983) and Taylor (1986), among others, emphasize the derivation of laws determining the city size distribution from some microeconomic principles. Lastly, at the system-level, researchers pay much attention to the behavior of the urban hierarchical system as a whole. As a result, the well-known rank-size rule and a family of skewed city-size distributions have been demonstrated in some earlier studies such as that of Beckmann (1958), Parr (1969) and Beguin (1979). It seems that at present we are unable to assert which level of analysis is superior to the others, because our knowledge about the urban hierarchy still remains very limited. Investigation at all the levels would undoubtedly enrich our understanding of the urban system.

By using the terms of the aforementioned classification, the present paper would probably be grouped into the settlement-level of analysis. That is, we want to show some economic principles determining the spatial structure, city size and city number, given the framework of a hierarchical inter-urban system. The detailed motivations of this work are as follows.

As is well known, in the existent literature of the settlement-level, given the context of a central place system, the equilibrium resulting from the behavior of profit-maximizing firms and utility-maximizing households have been thoroughly discussed, and the effects of scale economies, the structure of demands, utility equilibrium and other economic parameters on the determination of city size and city number have been rigorously examined. However, in these studies, the spatial factors (e. g., the location of economic actors) have received very little attention so far, and the spatial structure of the urban hierarchy has not yet been explained satisfactorily. It seems to us that the existing economic theory of urban hierarchy at the settlement-level needs to incorporate the new developments of urban land-use theory into consideration.

Turning to the field of urban land-use theory, we find that although there have been a large number of studies on the monocentric spatial structure since Alonso's seminal work (1964), the endogenously multicentric land-use model that was developed by Fujita and Ogawa (1982) proved to be very useful in explaining the spatial structure of the vast inter-urban space. Unfortunately, however, in their work, though the possible formation of the multicentric spatial configuration was demonstrated, the hierarchy of the urban system, which is one of the most important characteristics of the real urban world, has not been taken into account. To contribute to this point, by using a rather different framework, Zheng (1990) presents a model of an inter-urban system in which the urban hierarchy is explicitly considered. The focus of his paper is on the derivation of various land-use patterns from a specified two-rank city system, but the determination of city size and city number in a more general hierarchical urban system still remains to be studied.

Given shortcomings of earlier studies, the purpose of this paper is to investigate the principles that determine the spatial structure, city size and city number in the context of a generalized hierarchical inter-urban system. The system considered here is such an urban hierarchy in a one-dimensional region that has a tree-shaped structure, i. e., cities at any rank are supposedly influenced by the city at the next higher rank (but not all the cities at higher ranks). It is assumed that each city contains a business firm and a class of households employed by the firm. By supposing that the business firm at any city rank has to transport its intermediary inputs from the firm at the next higher rank, and that the firm needs to communicate with the households, we shall show that we can obtain a spatial equilibrium of the urban system that depends on the magnitude of transportation cost. As a result, the city size and city number will also be dependent upon the transportation and communication costs. By comparing this market solution to a socially optimal solution of city size and city number, we find that the equilibrium city size would be larger while the city number would be less than the optimal one throughout all the city ranks. This conclusion has some important implications for the urban growth control policies now widely implemented in practice.

The rest of the paper is organized as follows. In Section 2, a simple model of a hierarchical inter-urban system will be presented, whose equilibrium and optimal solutions will be demonstrated in Sections 3 and 4, respectively. Finally, the paper will be concluded in Section 5.

2. THE MODEL

2. 1. The hierarchical inter-urban system

Let us consider a long strip of homogeneous land upon which a hierarchical system of cities is to develop. The width of the strip is one unit of distance while its length is supposed to be sufficiently long. For simplicity, the structure of the urban hierarchy considered here is thought to be tree-shaped in that a city at any rank would dominate a number of cities at the next lower rank by providing intermediary inputs for the firms in those cities. Furthermore, to make things simple, it is assumed that such a domination only occurs between any two neighboring ranks of cities but not between other kinds of pairs of city ranks. Fig. 1 illustrates such an urban hierarchy, which has the same structure as that Isard showed in his famous book (Isard, 1975, Fig. 12. 1, p. 288). Each city consists of a business firm and a class of households that are employed by the firm. In the cities of the same rank, business firms and households are considered to be identical in terms of production technology and job suitableness. But they are different from those in the cities of other ranks. In particular, it can be considered that the rank of firms are mainly determined by the technologies that the firms possess, i. e., firms at a higher rank of cities usually have technology of higher productivity than those at lower ranks.

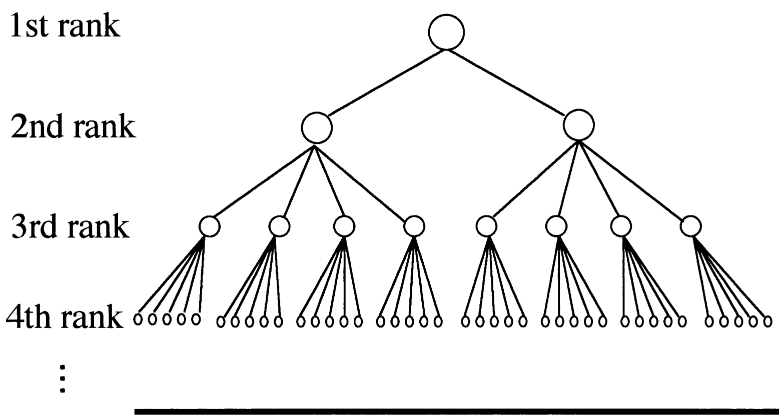

Fig. 1 The hierarchical inter-urban system

2. 2. Households

Suppose the number of types of households is the same as that of city ranks in the urban system. Each type of households is assumed to be distributed at a constant density, which may differ from that of other types. From the literature of location theory, we can see that such a uniform distribution of households is often assumed. (e. g., Hotelling, 1929)

It is assumed that households at any city rank are to be employed by the business firms at the same city rank. They will commute to work at the places where the firms are located and gain the wage as their only source of income. Using this income, they pay the commuting cost that is supposed to be proportional to the distance they commute over, and pay for the consumption of goods and services that are assumed to be imported from the outside markets. For simplicity, we assume throughout this paper that the households do not consume land, which needs to be relaxed in future studies.

2.3. Business firms

Like the households, we suppose that the number of types of business firms is the same as that of the city ranks in the linear region. It is assumed that a business firm at any city rank produces some export goods by using the intermediary inputs transported from the firms at the next higher city rank, and hiring the households at the same city rank.

Concerning the behavior of business firms, we shall propose a few additional assumptions as follows. Firstly, in buying the intermediary goods from firms at the higher city rank, besides the price, the firm has to pay the transportation cost that is assumed to be proportional to the distance between the two transacting firms. So, for a firm located at x, the cost of transporting intermediary inputs from a firm of higher city rank at z can be given by:

$$T(k,x)=k\,|\,x-z\,|S \qquad (2.1)$$

where k is the transportation cost per unit distance, and S the amount of intermediary goods purchased.

Secondly, as Zheng (1990) assumes, in employing households the firms need to communicate with them in order to find capable men and women for the work. Concerning such a communication between firms and households, it should be noted that because of technological progress, it becomes easier for the firm to get its intermediary inputs such as money and information, while human capital of high quality becomes relatively hard to obtain. For this reason, it was reported recently that there have been many Japanese giant companies (say Toyota and NEC) locating their plants and branches in a few peripheral areas. More specifically, such a communication in general bears a cost that has to be deducted from the firm's profit and is supposed to be proportional to the distance between firms and households. Thus, for a firm located at x, the total cost of communicating with the households living in segment [a, b] can be expressed by:

$$C(c,x)=c\int_a^b h(y)|y-x|\,dy \tag{2.2}$$

where c is the communication cost per unit distance, and $h(y)$ the density of households at y.

For a firm located at x, if N and S denote the amounts of labor (i. e., households) and intermediary goods inputted, respectively, its profit (π) can be written as:

$$\pi=PQ(N,S)-wN-qS-T(k,x)-C(c,x) \tag{2.3}$$

where $Q(N,S)$ is the firm's production function, p the price of export products, w the wage, and q the price of intermediary goods.

3. MARKET EQUILIBRIUM

3. 1. Additional assumptions

First of all, without loss of generality, we assume that the hierarchical inter-urban system considered here is formed in such a process that cities at the first (highest) rank appear at first, and cities of the second and other lower ranks then emerge in due order. Thus, if we could illustrate the formation of cities at the first and second ranks, the growth of cities at other lower ranks can be easily demonstrated in a similar fashion.

Since the structure of the urban system is tree-shaped, for simplicity, we can suppose additionally that at the first rank, there is only one city to be formed. As has been assumed in the last section, a business firm of the city needs to buy intermediary inputs from the firms at the next higher city ranks (which, in the case of the first rank, means the firms outside the linear region), and to communicate with the households of the same rank distributed uniformly along the long strip. So, to save the communication cost with all households along the region, the firm at the first city rank will choose to locate at the center of the region.

The question then left to be answered is, where will the firms at the second rank be located, how many households will they employ, and how many such firms will enter this linear region. For simplicity of analysis, we suppose that there will appear two identical series of such firms symmetrical about the regional center, and that the firms will be located in such an order that starts from the center to the end of the linear region. Furthermore, it is also assumed that the length of the strip is sufficiently long compared to the number of firms to be located. So, if the length is let be $2F$, it can be thought to be divided just by $2n$ firms of the same second city rank. Clearly, in the half strip on the right-hand side of the center (its length is F), there will be n firms (or cities) at the second city rank. In the following, we shall only show how the location of firms, the city size and city number on the right-hand side half strip are determined (see Fig. 2). The situation on the left-hand side will follow by a similar argument.

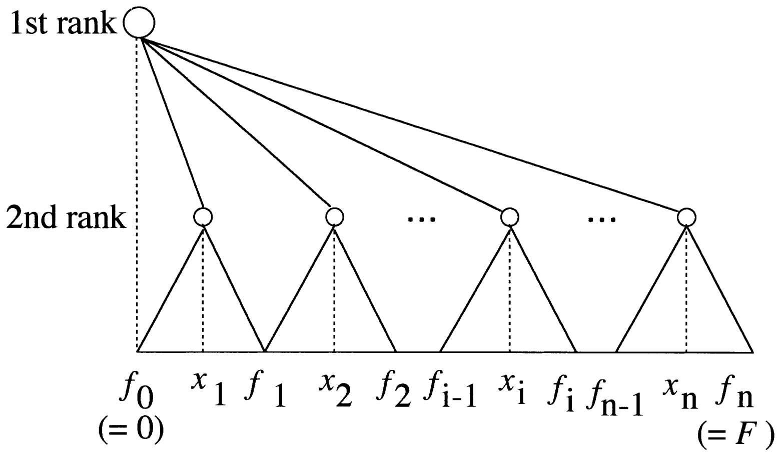

Fig. 2 The first and second ranks of cities
(only the right-hand side half strip is shown here)

Next, more specifically, we assume that the firms in question have fixed-coefficient production technology, i. e., their production function is given by:

$$Q(N, S)=\min(\alpha N, \beta S)+\gamma \tag{3.1}$$

where α, β and γ are positive parameters. Denote the variables of the ith firm at the second city rank by the subscript i, then according to eq. (2.3) the profit of the firm can be expressed by:

$$\pi_i=p(\alpha N_i+\gamma)-w_iN_i-(q+kx_i)\frac{\alpha}{\beta}N_i-c\int_{f_{i-1}}^{f_i}h\,|y-x_i|\,dy, \qquad i=1,\ 2,\ \cdots,\ n \tag{3.2}$$

where x_i is the location of the firm, h the fixed density of the households to be employed by the firms at the same rank, and f_{i-1} and f_i are boundaries of the area within which the households employed by the ith firm live. In other words, f_{i-1} and f_i also mean the boundaries of the ith city at the second rank. Since the coordinate of the regional center is zero, for the first firm (i=1) on the right-hand side of the center, we have $f_0=0$. So, for the ith firm,

f_{i-1}, which is also the outer boundary of the $(i-1)$th city, could be considered as given, but f_i is to be determined by:

$$f_i = f_{i-1} + \frac{N_i}{h} \tag{3.3}$$

Finally, concerning the prices of goods appearing in this model, we suppose that the products of firms are all exported to the markets outside the region, and that their prices, p, can be considered as exogenously given. As for the price of intermediary goods, q, since all the products are exported, and the goods will be purchased according to the outside market, it can also be treated as a constant.

3. 2. Equilibrium conditions

Under the situation described in the last subsection, let us think about the equilibrium conditions for the hierarchical inter-urban system in question.

Suppose that the firm is to maximize its profit by determining the amount of labor (households) and the place of its location. Here, the firm is not allowed to change the wage for competing with other firms for households, since it will make the problem too complicated to be solved. In doing so, we have the following two equilibrium conditions:

$$\frac{\partial \pi_i}{\partial N_i} = 0, \qquad i = 1, 2, \cdots, n \tag{3.4}$$

$$\frac{\partial \pi_i}{\partial x_i} = 0, \qquad i = 1, 2, \cdots, n \tag{3.5}$$

In addition, if we consider that the entry of firms into the linear region will continue until the firm's profit becomes zero, there will be a zero-profit condition in the long term as follows:

$$\pi_i = 0, \qquad i = 1, 2, \cdots, n \tag{3.6}$$

Now let us manipulate these conditions. Firstly, from eqs. (3.2) and (3.4) we get:

$$\frac{\partial \pi_i}{\partial N_i} = p\alpha - w_i - (q + kx_i)\frac{\alpha}{\beta} - c\left(f_{i-1} + \frac{N_i}{h} - x_i\right) = 0 \tag{3.7}$$

which yields:

$$w_i = p\alpha - (q + kx_i)\frac{\alpha}{\beta} - c\left(f_{i-1} + \frac{N_i}{h} - x_i\right) \tag{3.8}$$

By using eqs. (3.2) and (3.5), we obtain:

$$\begin{aligned}
\frac{\partial \pi_i}{\partial x_i} &= -k\frac{\alpha}{\beta}N_i - ch\frac{\partial}{\partial x_i}\int_{f_{i-1}}^{f_i} |y - x_i| dy \\
&= -k\frac{\alpha}{\beta}N_i - ch\frac{\partial}{\partial x_i}\left[\int_{f_{i-1}}^{x_i}(x_i - y)\,dy + \int_{x_i}^{f_i}(y - x_i)\,dy\right] \\
&= -k\frac{\alpha}{\beta}N_i - ch\left(2x_i - 2f_{i-1} - \frac{N_i}{h}\right) = 0
\end{aligned} \tag{3.9}$$

which gives the following expression:

$$x_i = f_{i-1} + \left(1 - \frac{\alpha k}{\beta c}\right)\frac{N_i}{2h} \tag{3.10}$$

Second, by substituting eq. (3.8) into eq. (3.2) we have:

$$\begin{aligned}\pi_i &= p\gamma - c\left(f_{i-1} + \frac{N_i}{h} - x_i\right)N_i - ch\int_{f_{i-1}}^{f_i} |y - x_i| dy \\ &= p\gamma - c\left(f_{i-1} + \frac{N_i}{h} - x_i\right)N_i - ch\left[\int_{f_{i-1}}^{x_i}(x_i - y)\,dy + \int_{x_i}^{f_i}(y - x_i)\,dy\right] \\ &= p\gamma - c\left(f_{i-1} + \frac{N_i}{h} - x_i\right)N_i - \frac{ch}{2}\left[\left(f_{i-1} + \frac{N_i}{h} - x_i\right)^2 + (x_i - f_{i-1})^2\right]\end{aligned} \tag{3.11}$$

which, by using eq. (3.10), will give:

$$\pi_i = p\gamma - \frac{c}{4h}\left[\left(\frac{\alpha k}{\beta c} + 1\right)^2 + 2\right]N_i^2 \tag{3.12}$$

So, from eq. (3.6) we get:

$$N_i = 2\sqrt{\frac{p\gamma h}{c\left[\left(\frac{\alpha k}{\beta c} + 1\right)^2 + 2\right]}} \tag{3.13}$$

Here, if we use eq. (3.3), we get the following:

$$f_i - f_{i-1} = \frac{N_i}{h} = 2\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c} + 1\right)^2 + 2\right]}} \tag{3.14}$$

Since in the last subsection we assumed that the length of the whole strip is $2F$ with its center as the origin, we have:

$$f_0 = 0 \tag{3.15}$$

$$f_n = F \tag{3.16}$$

The solution for the difference equation system composed by eqs. (3.14) to (3.16) will yield:

$$f_i = 2i\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c} + 1\right)^2 + 2\right]}}, \qquad i = 1, 2, \cdots, n \tag{3.17}$$

$$n = \frac{F}{2}\sqrt{\frac{ch\left[\left(\frac{\alpha k}{\beta c} + 1\right)^2 + 2\right]}{p\gamma}} \tag{3.18}$$

In short, from the equilibrium conditions we finally obtained a system of equations, i. e., (3.8), (3.10), (3.13), (3.17) and (3.18), for the following five unknowns, w_i, x_i, N_i, f_i and n ($i = 1, 2, \cdots, n$).

3. 3. Equilibrium properties

In this subsection, we shall show some important equilibrium properties of the hierarchical inter-urban system. In the first place, by solving the difference equation system obtained previously, we can express the equilibrium wage as follows:

$$w_i = p\alpha - q\frac{\alpha}{\beta} - c\left[2i\frac{\alpha k}{\beta c} - \left(\frac{\alpha k}{\beta c}\right) + 1\right]\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}}, \quad i=1,\ 2,\ \cdots,\ n \tag{3.19}$$

which yields:

$$\frac{dw_i}{di} = -2\frac{\alpha k}{\beta}\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}} < 0 \tag{3.20}$$

This means that in equilibrium, the wage paid by firms to households will depend on the location of firms. *The more distant the firms are located away from the regional center, the lower the wage they would pay to the households.* It should be noted that if the wage becomes negative, households may be better off by being unemployed. So, to avoid this possibility, we could assume that the price of export product (p) given by the outside market should be sufficiently large to ensure the wage be positive.

Concerning the location of firms, substitution of eqs. (3.13) and (3.17) into eq. (3.10) gives:

$$x_i = \left(2i - \frac{\alpha k}{\beta c} - 1\right)\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}}, \quad i=1,\ 2,\ \cdots,\ n \tag{3.21}$$

whose derivatives with respect to k and i are given as follows:

$$\frac{dx_i}{dk} = -\frac{2a}{\beta c}\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}}\left[\left(\frac{ak}{\beta c}+1\right)i+1\right] < 0 \tag{3.22}$$

$$\frac{dx_i}{di} = 2\sqrt{\frac{p\gamma}{ch\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}} > 0 \tag{3.23}$$

(3.22) implies that when the cost of transporting the intermediary goods (that is, some high-grade goods or services for the production activities, such as the information through face-to-face contacting with firms of higher city ranks) from the regional center increases, the firms tend to locate nearer to the center. And (3.23) means that firms at the same rank will be allocated in an order that starts from the center to the end of the linear region. Based on these results, it is easy to draw a group of x_i curves for different values of i on the plain of x and k, as is shown in Fig. 3. From this figure, we can see that *the spatial structure of the hierarchical inter-urban system is heavily influenced by the transportation cost of the intermediary goods or services.* That is, when the transportation cost (k^*) is high, most of the firms would locate within a limited area surrounding the center, and the spatial structure of such a limited space would become very concentrated. But, when the transportation cost is lowered due to some technological progress (say the development of new technology for contacting between firms), the firms would tend to locate in a dispersed manner in order to save the cost of communicating with the households distributed uniformly along the region.

As for the city size (i. e., the number of households employed by the firm) and the city

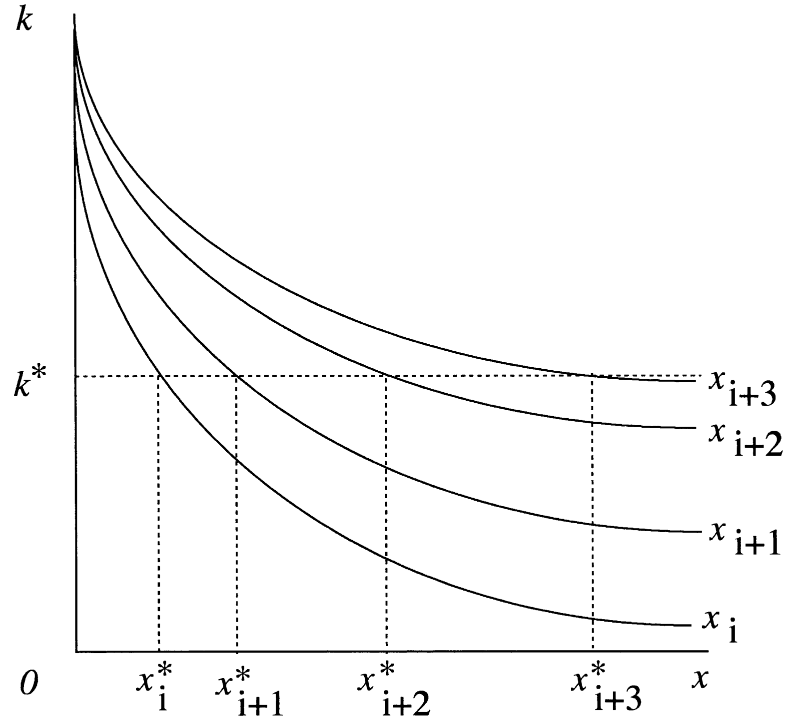

Fig. 3 Location of firms (x_i) and transportation cost (k)

number (the number of firms allocated into the region), according to eq.(3.13) and eq. (3.18), we can see that like the spatial structure, *both of the city size and city number are also dependent upon the magnitudes of transportation and communication costs (i.e., k and c).* This result, however, has not been pointed out by the existing economic theory of urban system (say Henderson, 1987), since it did not take into account the interactions between firms of different city ranks.

It should be noted that by looking at Fig. 3, one could find that there exists a boundary condition for the location of firms to be meaningful in the linear space, i. e.,

$$x_i \geq 0, \qquad i=1, 2, \cdots, n \tag{3.24}$$

Using eq. (3.21) we can rewrite it as follows:

$$2i - \frac{\alpha k}{\beta c} - 1 \geq 0, \qquad i=1, 2, \cdots, n$$

which is equivalent to the following expression

$$\frac{\alpha k}{\beta c} \leq 1 \tag{3.25}$$

This inequality could be considered as a sufficient condition for the spatial equilibrium to exist.

Finally, let us check the second-order conditions for the firm's profit-maximizing problem. The second derivatives of eq. (3.7) and eq. (3.9) yield the following expressions:

$$\frac{\partial^2 \pi_i}{\partial N_i^2} = -\frac{c}{h} \tag{3.26}$$

$$\frac{\partial^2 \pi_i}{\partial N_i \partial x_i} = \frac{\partial^2 \pi_i}{\partial x_i \partial N_i} = -k\frac{\alpha}{\beta} + c \tag{3.27}$$

$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -2ch \tag{3.28}$$

$$\begin{vmatrix} \dfrac{\partial^2 \pi_i}{\partial N_i^2} & \dfrac{\partial^2 \pi_i}{\partial N_i \partial x_i} \\ \dfrac{\partial^2 \pi_i}{\partial x_i \partial N_i} & \dfrac{\partial^2 \pi_i}{\partial x_i^2} \end{vmatrix} = \left[(\sqrt{2}+1)c - k\frac{\alpha}{\beta}\right]\left[(\sqrt{2}-1)c + k\frac{\alpha}{\beta}\right] \tag{3.29}$$

For such a profit-maximizing problem to have a maximum, the second-order matrix should be negative definite, which requires that the right-hand side of eq. (3.29) should be positive. By calculation, this second-order condition is equivalent to:

$$1 - \sqrt{2} < \frac{\alpha k}{\beta c} < 1 + \sqrt{2} \tag{3.30}$$

Since α, β, k and c are all positive parameters, (3.30) equivalently becomes:

$$0 < \frac{\alpha k}{\beta c} < 1 + \sqrt{2} \tag{3.31}$$

If (3.31) is combined with the sufficient condition (3.25), we can obtain the final sufficient condition for the spatial equilibrium to exist as follows:

$$0 < \frac{\alpha k}{\beta c} \leq 1 \tag{3.32}$$

which implies that for the defined hierarchical inter-urban system to reach a stable equilibrium, the parameters representing the firm's marginal productivities, and the costs of transportation and communication should satisfy the condition expressed by the inequality of (3.32).

4. SOCIAL OPTIMUM

It is always pointed out that the equilibrium realized by the market mechanism would not necessarily be optimal from the perspective of social welfare. In this section, we shall present a socially optimal solution for the defined hierarchical inter-urban system, and compare it with the equilibrium solution discussed so far.

4. 1. Optimal conditions

The social optimality considered here is defined as an allocation of regional resources that maximizes the total of social net profits from the whole urban system. The word "net" means that the cost of households' commuting should also be deducted from the business firms' profits, because it is one kind of social costs. So, by taking the cities at the second rank in the right-hand side half strip of the region as an illustration, as we did in the last section, the optimality can be expressed by the following maximization expression:

$$\left\{\begin{array}{l} \max \phi=\sum_{i=1}^{n}\lambda_i\Big[p(\alpha N_i+\gamma)-w_iN_i-(q+kx_i)\dfrac{\alpha}{\beta}N_i-c\displaystyle\int_{f_{i-1}}^{f_{i-1}+N_i/h}h|y-x_i|dy \\ \qquad -t\displaystyle\int_{f_{i-1}}^{f_{i-1}+N_i/h}h|y-x_i|dy\Big] \\ \text{with respect to} \qquad N_i,\ x_i,\ \lambda_i\ (i=1,\ 2,\ \cdots,\ n) \end{array}\right. \tag{4.1}$$

where ϕ is the total of social net profits, λ_i the shadow price of the ith city, and t the commuting cost per unit distance.

The first-order conditions for the above mentioned maximization problem give:

$$\frac{\partial\phi}{\partial N_i}=\lambda_i\Big[p\alpha-w_i-(q+kx_i)\frac{\alpha}{\beta}-(c+t)(f_{i-1}+\frac{N_i}{h}-x_i)\Big]=0 \tag{4.2}$$

$$\frac{\partial\phi}{\partial x_i}=\lambda_i\Big[-k\frac{\alpha}{\beta}N_i-(c+t)h\frac{\partial}{\partial x_i}\int_{f_{i-1}}^{f_{i-1}+N_i/h}|y-x_i|dy\Big]=0 \tag{4.3}$$

$$\frac{\partial\phi}{\partial x_i}=p(\alpha N_i+\gamma)-w_iN_i-(q+kx_i)\frac{\alpha}{\beta}N_i-(c+t)\int_{f_{i-1}}^{f_{i-1}+N_i/h}h|y-x_i|dy=0 \tag{4.4}$$

From eq. (4.2), we have:

$$w_i=p\alpha-(q+kx_i)\frac{\alpha}{\beta}-(c+t)(f_{i-1}+\frac{N_i}{h}-x_i) \qquad i=1,\ 2,\ \cdots,\ n \tag{4.5}$$

which is a condition concerning the wage level that corresponds to eq. (3.8) in the equilibrium. By calculating eq. (4.3), we obtain the following expression:

$$-k\frac{\alpha}{\beta}N_i-(c+t)h(2x_i-2f_{i-1}-\frac{N_i}{h})=0 \tag{4.6}$$

which yields for x_i:

$$x_i=f_{i-1}+\Big[1-\frac{\alpha k}{\beta(c+t)}\Big]\frac{N_i}{2h}, \qquad i=1,\ 2,\ \cdots,\ n \tag{4.7}$$

(4.7) is in fact the counterpart of the equilibrium condition of eq. (3.10).

Substitution of eqs. (4.5) and (4.7) into eq. (4.4) yields:

$$p\gamma-\frac{c+t}{4h}\Big\{\Big[\frac{\alpha k}{\beta(c+t)}+1\Big]^2+2\Big\}N_i^2=0 \tag{4.8}$$

which is equivalent to:

$$N_i=2\sqrt{\frac{p\gamma h}{(c+t)\Big\{\Big[\dfrac{\alpha k}{\beta(c+t)}+1\Big]^2+2\Big\}}} \tag{4.9}$$

Here, by using eqs. (3.3), (3.15) and (3.16) we obtain:

$$f_i=2i\sqrt{\frac{p\gamma}{(c+t)h\Big\{\Big[\dfrac{\alpha k}{\beta(c+t)}+1\Big]^2+2\Big\}}}, \qquad i=1,\ 2,\ \cdots,\ n \tag{4.10}$$

$$n=\frac{F}{2}\sqrt{\frac{(c+t)h\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}}{p\gamma}}, \tag{4.11}$$

In this way, from the first-order conditions we obtained a system of difference equations, (4.5), (4.7), (4.9), (4.10) and (4.11), for solving the following five variables, w_i, x_i, N_i, f_i and n ($i=1, 2, \cdots, n$).

4. 2. Comparison between equilibrium and optimum

Let us compare the solutions for city size and city number between the market equilibrium and the social optimum. Let N_i^{eq} and N_i^{opt} denote the equilibrium and optimal solutions for the city size, respectively. Using eqs. (3.13) and (4.9), we have:

$$N_i^{eq}-N_i^{opt}=2\sqrt{p\gamma h}\left\langle\frac{1}{\sqrt{c\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]}}-\frac{1}{\sqrt{(c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}}}\right\rangle \tag{4.12}$$

from which a comparison of the denominators in the angle bracket yields:

$$\begin{aligned}\Delta&\equiv(c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}-c\left[\left(\frac{\alpha k}{\beta c}+1\right)^2+2\right]\\&=t\left[3-\frac{\alpha^2k^2}{\beta^2c(c+t)}\right]\end{aligned} \tag{4.13}$$

By using the second-order condition of eq. (3.29), we get:

$$\Delta=t\left[3-\frac{\alpha^2k^2}{\beta^2c(c+t)}\right]>t\left[1-\left(\frac{\alpha k}{\beta c}\right)^2\right]>0 \tag{4.14}$$

Thus, eq. (4.12) becomes:

$$N_i^{eq}-N_i^{opt}>0 \tag{4.15}$$

which means that the equilibrium city size is larger than the optimal one.

Next, by letting n^{eq} and n^{opt} be the equilibrium and optimal solutions for the city number, respectively, from eq. (3.18), eq. (4.11) and the inequality of (4.14) we can obtain the comparison as follows:

$$n^{eq}-n^{opt}=\frac{F}{2}\sqrt{\frac{h}{p\gamma}}\left\langle\sqrt{c\left[\left(\frac{\alpha k}{\beta c}\right)^2+1\right]}-\sqrt{(c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}\right]^2+1\right\}}\right\rangle<0 \tag{4.16}$$

That is, the equilibrium city number is less than the optimal one.

The inconsistency between the equilibrium and optimal solutions for the city size and city number seems to result from the commuting cost of households. In the market equilibrium, unlike in the social optimum, the households' commuting cost in general does not need to be considered in the firms' profit-maximizing behavior, so, by this saving, the firms can employ a few more households that causes the equilibrium city size to be larger than the optimal one. At the same time, due to the limited length of the region, larger city size means that a smaller number of cities could enter the space. Thus, the equilibrium city number will turn out to be less than the social optimum.

This result can be easily applied to the other city ranks of the urban system. In other words, *throughout all the city ranks, by the market mechanism, the city size would be too large while the city number would be too small when compared to the social optimum.* The policy implication from this conclusion is that in the real urban world, to realize such a socially optimal urban system, we should control the possible excessive city sizes and meanwhile increase the likely smaller number of cities at all city ranks of the urban system.

This kind of urban growth control policies, in fact, have already been implemented in practice. For example, in urban Japan, as the Tokyo metropolitan area becomes larger and complex, many urban policies have been proposed to control further expansion of the central part of Tokyo, and to promote growth of the peripheral cities around it. This means that the excessive size of the central city is under the control while a few peripheral cities are to be created. As we have illustrated, these policies can find a theoretical basis from the development of the urban system theory.

5. CONCLUDING REMARKS

In this paper, we presented a simple model of a hierarchical inter-urban system, which is supposed to be tree-shaped, containing a few ranks of cities constituted by differentiated business firms and households. By assuming that the firm at any city rank has to pay both the cost of transporting its intermediary inputs supplied from the firm at the next higher rank, and the cost of communicating with the households at the same rank, we showed that there would exist an equilibrium of spatial structure in which the location of firms may be concentrated or dispersed, depending on the transportation cost. It was also demonstrated that in the equilibrium, the resulted city size and city number at each rank are dependent upon the magnitude of transportation and communication costs with which the firms are confronted. Such an equilibrium solution, however, would not necessarily be socially optimal. In fact, we found that by the market principle, the city size would be too large while the city number would be too small when compared to the social optimum. This kind of possibility of excessive city size has once been pointed out before by a few urban economists, e. g. Henderson (1987), but we want to emphasize that the framework of theorizing used here is very different in that the transaction amongst cities as well as the hierarchical property of the urban system are explicitly taken into account in the present study.

This work might be considered as a generalization of Zheng's model (1990) in the sense that it is applied to a hierarchical inter-urban system with more than two ranks of cities. But, at the price of such a generalization, we neglected the land market, assuming that all the types of households are uniformly distributed along the linear region. As the result, the aforementioned conclusions seem to be dependent to a great extent on these unrealistic assumptions. For example, the inconsistency between the equilibrium and optimal solutions may be partly because that the behavior of households and the resulted equilibrium were not analyzed here. More specifically, if the land market could be considered in the paper, the commuting cost that leads to this inconsistency would be offset to some extent by the resulted equilibrium land rent. However, introducing the land market into the context of firm-location models would make the analysis very complicated, which seems beyond the main purpose of this paper. Besides, for simplicity of illustration, we discussed the spatial structure, city size and city number of the hierarchical inter-urban system, but without mentioning the formation and evolution of such an urban hierarchy. In addition, it should also be noted that in this paper the size and number of a city were represented by that of a firm, and the city was considered as a company town. Strictly speaking, a city never means a firm only. Rather, a variety of firms and households constitute the city, which should be taken into account in the future. So, the next step we should take is to make the present work more general and reasonable. Furthermore, as another direction for further extensions, as Zheng (1991, 1998) shows, we need to carry out an empirical study on some metropolitan areas in the real world to test the existing urban system theory, since, for the present, much less rigorous empirical studies of the urban system appear in the literature than related theoretical works.

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