Durable Goods Monopoly and Quality Choice: Selling vs. Renting

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1. Introduction

The literature related to durable goods markets concludes that a monopolist who sells durable goods builds less than the socially optimal level of physical durability into its output; that is, it practices planned obsolescence (for example, see Coase (1972), Bulow (1982, 1986) and Bond and Samuelson (1984)). The monopoly seller can, however, also introduce new products that may make consumers possessing the old units replace them. Kinokuni (2000) develops a model of a durable goods monopoly seller who can improve product quality over time and endogenously determines the quality of the new product. The paper shows that the monopoly seller who faces the time inconsistency problem has an incentive to produce additive units ex post and chooses a quality level which is below the socially optimal level. The present paper extends the Kinokuni's model and compare the monopolist's quality choice in the sales case with that in the rental case. It is important to distinguish between the sales contract and the rental contract in considering a durable goods market. When the monopolist sells durable goods, it faces the situation of the Coase conjecture, that is, it sells durable goods at the competitive price and no gains. If the monopolist could rent durable goods, it can overcome the Coase conjecture and gain the non-durable goods monopolist's profits. Therefore, the U.S. government has required the dominant firm in some industries (computers, copies and shoe machinery) to sell rather than rent by the reason that the rental policy brings the anticompetitive outcome into the market.

This paper demonstrates that, in the case where the unit cost is sufficiently elastic with respect to quality, the monopoly seller chooses a higher quality than the monopoly renter chooses. Therefore, in this case, prohibiting the monopolist from renting its product to consumers is appropriate from the viewpoint of the quality improvement. In the case where the unit cost is not sufficiently elastic with respect to quality, the monopoly seller chooses a lower quality than the monopoly renter chooses. In this case, prohibiting the rental policy makes the monopolist choose a lower product quality. It is also shown that the existence of transaction costs in the secondhand market reduces the monopoly seller's incentive to improve product quality.

Although there is no literature dealing with the quality improvement activity in the rental case, there is a growing body of research related to the durable goods monopoly seller's intro-
duction of new products. Waldman (1993) and Choi (1994) construct the model of a durable goods monopoly seller characterized by network externalities and show that the monopolist has a too high incentive to introduce new products that are incompatible with previous units. Waldman (1996) shows that a level of R & D a monopoly seller of durable goods chooses may be less than the socially optimal level. There are three important differences between Waldman's model and this paper's. First, Waldman assumes that there is the uncertainty of whether the R & D investment will be successful or not and the quality level of new output is exogenous. In this paper, assuming that the R & D investment is successful, a monopolist endogenously determines the quality level of the new output. Second, Waldman assumes there are two types of consumers. This paper assumes, however, there are a continuous range of consumers who differ by tastes. Third, this paper compares the monopoly seller's incentive to improve its product quality with the monopoly renter's one.

The outline for the paper is as follows: Section 2 outlines our basic model of a durable goods monopoly and derives the conditions for the level of quality a monopoly seller and a monopoly renter choose. Section 3 compares the level of quality a monopoly seller chooses with that a monopoly renter chooses. Section 4 extends the basic model and analyzes the effect of the existence of transaction costs in the secondhand market on the quality level the monopoly seller chooses. Our results are summarized in Section 5.

2. The Basic Model

2.1. Assumptions

This paper considers a durable goods monopoly using a two-period model. We investigate two cases; the monopoly seller case and the monopoly renter case. The monopolist can improve product quality by engaging in R & D in period 2. It is assumed that the quality of the second-period output is higher than that of the first-period output. In this section, for simplicity, the output is assumed to be perfectly durable. In period 1, the firm chooses a quantity \( q_1 \). Let \( s_1 \) represent product quality of the output produced in period 1. Assume that \( s_1 \) is given. In period 2, the monopolist chooses both a quantity \( q_2 \) and a quality \( s_2 \), where \( s_2 > s_1 \). We can also write each period's product quality as

\[
s_1 = s \quad \text{and} \quad s_2 = k \times s,
\]

where \( s \) is positive and constant, and \( k \ (> 1) \) denotes the rate of improvement of product quality. With no loss of generality, it is assumed that \( s = 1 \). Therefore, the second-period quality choice is equivalent to the choice of \( k \). In period 2, old units (quality \( s_1 = 1 \)) and new units (quality \( s_1 = k \)) have the same generic type, but are vertically differentiated from each other.

In period \( i \) (\( i = 1, 2 \)), type \( \theta \) consumer derives a gross benefit \( \theta s_i \) from the consumption of a unit of quality \( s_i \), where \( \theta \) indexes a consumers’ taste parameter that is distributed uniformly on the interval \([\theta, \bar{\theta}]\). That is, \( \theta s_i \) is type \( \theta \) consumer’s valuation of the gross services provided by a unit of quality \( s_i \) for one period. Consumers' net utility is the present discounted value of the total service that the output yields over its lifetime minus the price \( P_i \). Each consumer
consumes either zero or one unit of the output.

The first-period unit production costs are given by \( c_1 \), where \( c_1 \) is positive and constant. The second-period's unit production costs are given by \( c(k) \), where

\[
    c'(k) > 0 \quad \text{and} \quad c''(k) > 0 \quad \text{for all} \quad k > 1.
\]

(2)

With no loss of generality, it is assumed that there are no fixed costs. The discount factor is \( 0 < \delta \leq 1 \). We will also assume the values of all parameters are common knowledge.

2. 2. Monopoly seller's quality choice

In this subsection, we consider a monopolist who sells durable goods and derive the equilibrium conditions for quality choice. When consumers who possess an old product purchase a new one, they have an incentive to resell the old unit to consumers who do not possess it. This transaction occurs in the secondhand market which is assumed to be competitive. We also assume that the monopoly seller can not price discriminate due to the possibility of arbitrage. That is, in the beginning of the second period the monopolist can not sell the new product at different prices to those consumers who possess and those who do not possess the old product.

First, we will examine the incentive constraints of consumers in period 1. In period 1, the incentive constraints of consumers who purchase a unit of quality \( s_1 \) at the price \( P_1 \) is

\[
    \theta s_1 + \delta P_{SH} - P_1 \geq 0,
\]

where \( P_{SH} \) denotes the price of a unit of quality \( s_1 \) charged in the secondhand market. Thus, consumers of type \( \theta \geq \theta_1 \equiv (P_1 - \delta P_{SH}) / s_1 \) purchase a unit of quality \( s_1 \) in period 1.

Next, we will consider the incentive constraints of consumers in period 2. We denote consumers who purchased the output in period 1 as 'consumers 1', and the remaining consumers as 'consumers 2'. Of consumers 2, the taste parameter of the consumer who is indifferent between a new unit of quality \( s_2 \) and a secondhand unit of quality \( s_1 \) is \( \theta_{ON} \equiv (P_2 - P_{SH}) / (s_2 - s_1) \). Consumers 2 of type \( \theta > \theta_{ON} \) prefer a new unit to a secondhand unit, and conversely. However, consumers 2 of type \( \theta < \theta_0 \equiv P_2 / s_2 \) will never purchase a new unit and those of type \( \theta < \theta_0 \equiv P_{SH} / s_1 \) will never purchase a secondhand unit.

In period 2, consumers 1 who purchased a unit of quality \( s_1 \) in period 1 have two options: to hold on to the old unit or resell it to consumers 2 of type \( \theta > \theta_0 \) at the price \( P_{SH} \) and purchase a new unit of quality \( s_2 \) at the price \( P_2 \). Of consumers 1, the taste parameter of consumer who is indifferent between the two options is \( \theta_{RS} \equiv (P_2 - P_{SH}) / (s_2 - s_1) \). Consumers 1 of type \( \theta > \theta_{RS} \) replace an old unit with new one, and those of type \( \theta < \theta_{RS} \) continue to possess old one. Note that \( \theta_{RS} = \theta_{ON} \).

We will investigate the relationship between consumers' taste parameter and each period's quantity produced. First, Figure 1 illustrates the case \( \theta_1 \geq \theta_{ON} = \theta_{RS} \). The quantity traded in the secondhand market is represented by \( q_{SH} \). In this case, \( \theta_0 < \theta_2 < \theta_{ON} \) necessarily holds, because consumers 1 have to charge a price that satisfies \( \theta_0 < \theta_{ON} \) (i.e., \( P_{SH} < (s_1 / s_2) P_2 \)) in order to resell the old unit in the secondhand market in period 2. Recalling that the consumers' taste parameter is distributed uniformly, each period's demand function is, respectively,
In this case, all of consumers 1 purchase a new unit in period 2, and the supplied quantity of the old unit is $\bar{\theta} - \theta_1$ and the quantity demanded is $\theta_{ON} - \theta_0$. Thus, the clearing price in the secondhand market is given by

$$P_{SH} = \frac{(s_2 - s_1) P_1 + s_1 P_2 - s_1 (s_2 - s_1) \bar{\theta}}{s_2 + \delta (s_2 - s_1)}.$$  

(5)

Next, Figure 2 illustrates the case $\theta_1 < \theta_{ON} = \theta_{eq}$. In this case, some consumers 1 purchase the new unit and none of consumers 2 purchase it. However, each period's demand function and
the price of an used good are the same as the case $\theta_i \geq \theta_{ON} = \theta_{RS}$. The case $\theta_i < \theta_0$ is impossible, because the price of the used good must satisfy $\theta_i \geq \theta_0$.

Substituting (5) into (3) and (4), simplifying by using (1), and rearranging yields each period's inverse demand function:

\[ P_1(q_1, q_2) = (1+\delta)\bar{\theta} - (1+\delta)q_1 - \delta q_2, \tag{6} \]
\[ P_2(q_1, q_2, k) = k\bar{\theta} - q_1 - kq_2. \tag{7} \]

Since consumers anticipate future prices based on present actions, a monopoly seller of durable goods must solve a dynamic program. Therefore, the problem of the monopoly seller is solved recursively. Given the first-period choice of $q_1$, the seller's second-period problem is given by

\[ \max_{q_2, k} \pi^S_2(q_1, q_2, k) = \{ P_2(q_1, q_2, k) - c(k) \} q_2. \]

Maximization of the second-period profits with respect to $q_2$ and $k$, respectively yields the first-order conditions:

\[ q_2 = \frac{k\bar{\theta} - c(k) - q_1}{2k}, \tag{8} \]
\[ \bar{\theta} - q_2 = c'(k). \tag{9} \]

We assume $q_2 > 0$ and $k > 1$. Although the sufficient conditions for profit maximization will be discussed in Section 3, here the sufficient conditions are assumed to be satisfied.

Combining conditions (8) and (9) yields the condition for a monopoly seller's quality choice:

\[ \bar{\theta} + \frac{q_1^S}{k} = 2c'(k) - \frac{c(k)}{k}. \tag{10} \]

Let $k^S$ denote the solution to condition (10). The first-period output level $q_1^S$ is obtained by solving the maximization problem of the first-period present discounted value of the firm's profit stream:

\[ \max_{q_1} \Pi^S(q_1, q_2(q_1), k(q_1)) = \{ P_1(q_1, q_2(q_1)) - c \} q_1 + \delta \{ P_2(q_1, q_2(q_1), k(q_1)) - c(k(q_1)) \} q_2. \]

The reaction function $k = k(q_1)$ is derived from condition (10), and the reaction function $q_2 = q_2(q_1)$ is derived from conditions (9) and (10).

2. 3. Monopoly renter's quality choice

The monopoly renter can maximize its profits in each period since it has the ownership of goods and there is no time inconsistency problem. The renter's problem in period 1 is given by

\[ \max_{q_1} \pi^R_1 = [\bar{\theta} - q_1 - c] q_1. \]

Thus, the first-period output level is $q_1^R = (\bar{\theta} - c) / 2$.

We now consider the renter's maximization problem in period 2. Let $P^R_0$ represent the old unit's price in period 2, and $P^R_2$ represent the new unit's price in period 2. Let $q_0$ denote the quantity of the old unit supplied in period 2, and $q_2$ denote the output level of the new unit in
period 2:
\[ q_0 = \frac{P_2^R - P_0^R}{s_2 - s_1} - \frac{P_0^R}{s_1}, \]
\[ q_2 = \theta - \frac{P_2^R - P_0^R}{s_2 - s_1}. \]

Therefore, the second-period inverse demand functions are, respectively,
\[ P_0^R (q_0, q_2) = \theta - q_0 - q_2, \]
\[ P_2^R (q_0, q_2, k) = k\theta - q_0 - kq_2. \]

Thus, the monopoly renter's second-period maximization problem is
\[ \max_{q_0, q_2, k} \pi_2^R = \left[ P_2^R (q_0, q_2, k) - c(k) \right] q_2 + P_0^R (q_0, q_2) q_0 \]
subject to \( q_0 \leq \frac{\theta - c(k)}{2} \).

Maximizing the second-period profits with respect to \( q_2 \) and \( k \) yields as the first order conditions:
\[ q_2 = \frac{k\theta - c(k) - 2q_0}{k}, \quad (11) \]
\[ \bar{o} - q_2 = c'(k). \quad (12) \]

Again, we assume \( q_2 > 0 \) and \( k > 1 \) and that the sufficient conditions for profit maximization are satisfied. Combining conditions (11) and (12) yields the condition for the renter's quality choice:
\[ \frac{k\theta - c(k) - 2q_0}{k} = 2c'(k) - \frac{c(k)}{k}. \quad (13) \]

Let \( k^R \) denote the solution to condition (13). The triplet \( (q_0^R, q_2^R, k^R) \) maximizes the renter's profits in period 2. If \( q_0^R > 0 \), then \( (q_0^R, q_2^R, k^R) \) is the solution to conditions (11), (12) and
\[ \bar{o} - 2q_0 - 2q_2 = 0. \]

The size of \( q_2^R \) depends on \( q_2^R \) and \( k^R \), therefore, it depends on the second-period unit production costs. Denoting \( \varepsilon (k) \) as the elasticity of unit cost with respect to quality, i.e.,
\[ \varepsilon (k) \equiv \frac{k c'(k)}{c(k)}. \]

The following proposition tells us the renter's marketing strategy concerning old goods.

**Proposition 1:** The monopoly renter of durable goods chooses \( q_0^R = 0 \) if \( \varepsilon (k^R) \geq \frac{3}{2} \) and \( q_0^R > 0 \) if \( \varepsilon (k^R) < \frac{3}{2} \).

**Proof.** The first-order derivative of \( \pi_2^R \) with respect to \( q_0 \) is
\[ \frac{\partial \pi_2^R}{\partial q_0} = \bar{o} - 2q_0 - 2q_2. \quad (14) \]
Substituting (11) and (12) into (14) and evaluating at \( q_0 = 0, q_2 = q_2^R \) and \( k = k^R \) and yields
Since the derivative (15) is non-positive if $e(k^g) \geq -\frac{3}{2}$, the monopoly renter chooses $q^g_0 = 0$, which maximizes the monopoly renter's profits. Since the derivative (15) is positive if $e(k^g) < -\frac{3}{2}$, the monopoly renter chooses

$$q^g_0 \in \left(0, \frac{\theta - c_1}{2}\right].$$

Q. E. D.

From proposition 1, the monopoly renter rents only new units in period 2 if $e(k) \geq \frac{3}{2}$ for all $k > 1$ and rents both old and new units if $e(k) < \frac{3}{2}$ for all $k > 1$. For example, if the unit cost function is $c(k) = \gamma k^\alpha$ where $\alpha$ and $\gamma$ are positive constant, the elasticity of the unit cost is constant, i.e., $e(k) = \alpha$.

3. The Analysis

We will denote the right hand side of conditions (10) and (13) by $F(k)$; i.e.,

$$F(k) = 2c'(k) - \frac{c(k)}{k}. \quad (16)$$

We assume that the unit cost function has the following characteristics:

$$F'(k) = \frac{1}{k^2} \left(2k^2 c''(k) - k c'(k) + c(k)\right) > 0 \text{ for all } k > 1. \quad (17)$$

Assumption (17) insures that $k^s$ (respectively, $k^g$) obtained by condition (10) (respectively, (13)) maximizes the monopoly seller's second-period profits (respectively, the monopoly renter's second-period profits). The inequality (17) states that twice the derivative of the marginal cost with respect to $k$ exceeds the derivative of the average cost with respect to $k$. Moreover, the condition

$$F(1) = 2c'(1) - c(1) < \bar{\theta} \quad (18)$$

insures that $k^s$ and $k^g$ exceed 1.

By comparing the level of quality chosen by a monopoly seller with one chosen by a monopoly renter, we obtain the following proposition.

**Proposition 2:**

(i) If $e(k^g) \geq -\frac{3}{2}$, then a monopoly seller chooses a higher product quality than a monopoly renter chooses.

(ii) If $e(k^g) < -\frac{3}{2}$, then a monopoly seller chooses a higher product quality than a monopoly renter chooses.
renter chooses when $q^s_i > 2q^R_o$, and the seller chooses a lower product quality than a monopoly renter chooses when $q^s_i < 2q^R_o$.

Proof.

(i) Since proposition 1 shows that $q^R_o = 0$ when $\varepsilon (k^R) \geq \frac{3}{2}$, the left hand side of condition (i) is larger than that of condition (ii) as long as $q^s_i > 0$. Therefore, we can conclude $k^s > k^R$ since $F^\prime (k) > 0$ (assumption (ii)).

(ii) When $\varepsilon (k^R) < \frac{3}{2}$, $q^R_o > 0$ from proposition 1. Comparing the left hand side of condition (ii) with that of condition (i) yields the following relationship:

$$q^s_i \equiv 2q^R_o \iff k^s \equiv k^R.$$

Q. E. D.

From the monopoly seller’s condition for quality choice (i) and the monopoly renter’s condition for quality choice (ii), the quality choices depend on the quantity of the old units supplied in period 2 in both cases. Thus, the firm who faces competition with the old product market has the incentive to offer the higher quality in order to raise the new product’s price. That is, the firm has the incentive to produce a more differentiated product in order to reduce the degree of competition with old units. In the sales case, the seller’s introduction of the new product makes consumers replace an old unit with a new one, and the secondhand market develops. In the rental case, there does not exist the secondhand market. The renter, however, chooses the quantity of old units which maximizes its second-period profits. If $\varepsilon (k^R) \geq \frac{3}{2}$, that is, the unit cost is sufficiently elastic with respect to quality, then the monopoly renter has an incentive to increase the quantity of the new product as a substitute for improving product quality. This eliminates the old product market. Therefore, the level of quality the monopoly renter chooses is lower than that the monopoly seller chooses. If $\varepsilon (k^R) < -\frac{3}{2}$, that is, the unit cost is not sufficiently elastic with respect to quality, then the monopoly renter has an incentive to improve the product quality as a substitute for increasing the quantity of the new product. Thus, the relationship between the quality levels chosen by the seller and the renter depends on the relationship between quantities of old units supplied in period 2. The reason for comparing $q^s_i$ and $2q^R_o$ is that the former constrains $q_o$ by affecting the new unit’s price, and the latter constrains it by affecting both old and new unit’s prices.

The result of proposition 2 suggests the following. In the case where the unit cost is sufficiently elastic with respect to quality, prohibiting the monopolist from renting its product to consumers is appropriate from the viewpoint of the quality improvement. However, in the case where the unit cost is not sufficiently elastic with respect to quality, prohibiting the rental policy may reduce the monopolist’s incentive to improve the product quality.
4. The Existence of Transaction Costs

The interesting point of the result obtained using our model is that the level of \( k^5 \) depends upon the quantity traded in the secondhand market. Therefore, a factor which relaxes the intensity of competition between the monopoly seller and the secondhand market also results in a lower level of \( k^5 \). In this section, we extend our model to consider the effect of the existence of transaction costs in the secondhand market on the monopoly seller's quality choice.

In the case of \( \theta_1 \geq \theta_{on} = \theta_{KS} \), the monopoly seller's introduction of the new product makes all consumers who possess the old unit replace it with new one. However, in practice, some of the consumers may choose not to replace due to the existence of transaction costs. For example, the consumer who is willing to replace the old product with new one may incur the costs of searching for a consumer who is willing to purchase the secondhand product, or the transportation costs. Moreover, the consumer who is willing to resell the old unit may incur the costs of verifying that the old unit is still in working condition. These transaction costs impede trades in the secondhand market, and dissuade consumers from replacing. In the following, we extend the basic model to consider the existence of transaction costs in the secondhand market.

For simplicity, the ratio of replacement purchases \( t \in [0, 1] \) is introduced as a proxy for transaction costs. The ratio of replacement purchases \( t \) measures the ratio of consumers who replace the old product with new one in period 2 to those who purchased the output in period 1, and it is exogenously given. The case \( t=1 \) is equivalent to the basic model where all of consumers 1 replace. The case \( t=0 \) is the case where none of consumers 1 replace. In the rental case, we does not need to consider such transaction costs, since there does not exist a secondhand market.

We now investigate the effect of the ratio of replacement purchases on the monopoly seller's quality choice. Incorporating the ratio of the replacement purchases \( t \) into the basic model, and rewriting each period's demand function respectively yields

\[
q_1 = \bar{\theta} - \frac{P_1 - \delta P_{SH}}{s_1},
\]

\[
q_2 = t\left( \bar{\theta} - \frac{P_1 - \delta P_{SH}}{s_1} \right) + \left( \frac{P_1 - \delta P_{SH}}{s_1} - \frac{P_2 - P_{SH}}{s_2 - s_1} \right).
\]

The secondhand market's equilibrium condition is given by

\[
t\left( \bar{\theta} - \frac{P_1 - \delta P_{SH}}{s_1} \right) = \frac{P_2 - P_{SH}}{s_2 - s_1} - \frac{P_{SH}}{s_1}.
\]

From the last three equations, the each period's inverse demand functions are

\[
P_1^T(q_1, q_2) = (1 + \delta) \bar{\theta} - (1 + \delta) q_1 - \delta q_2,
\]

\[
P_2^T(q_1, q_2, k) = k \bar{\theta} - (k - t(k - 1)) q_1 - k q_2.
\]

The monopoly seller's problem is given by

\[
\max_{q_1} \Pi^5 = [P_1^T(q_1, q_2) - c_1] q_1 + \delta [P_2^T(q_1, q_2, k) - c(k)] q_2
\]
subject to $\arg\max_{q_1, q_2, k} \pi^S_T = [P^T_2(q_1, q_2, k) - c(k)]q_2$ \(19\)

Let $(q^S_T, q^S_T, k^S_T)$ denote the solution to problem 19.

**Proposition 3**: If $q^S_T < 2q^R_0$, then a monopoly seller chooses a lower quality than a monopoly renter chooses. If $q^S_T > 2q^R_0$, then a monopoly seller chooses a lower quality than a monopoly renter chooses for $t \in [0, \tilde{t})$, and it chooses a higher quality than a monopoly renter chooses for $t \in (\tilde{t}, 1]$, where

$$
\tilde{t} = \frac{1}{1 + k^S_T} \left( k^S_T + \frac{2q^R_0}{q^S_T} \right).
$$

**Proof**. Following the procedure used in section 3 yields the monopoly seller’s equilibrium condition for quality choice, that is,

$$
\bar{\theta} + \left[ \frac{1}{1 + \frac{1}{k}} - 1 \right] q^S_T = 2c'(k) - \frac{c(k)}{k}.
$$

Comparing condition 20 with the condition for the monopoly renter’s quality choice 13 yields the above proposition. \(\text{Q. E. D.}\)

If $q^S_T < 2q^R_0$, then $k^R > k^S_T$ regardless of the ratio of replacement purchases. If $q^S_T > 2q^R_0$, then the relationship between $k^R$ and $k^S_T$ depends on the ratio of replacement purchases. The lower the ratio of replacement purchases leads to a reduction in the quantity traded in the secondhand market, and relaxes the intensity of competition between the monopoly seller and the secondhand market. As the ratio of the replacement purchases becomes lower, the seller must sell a larger proportion of the new product to consumers who have lower taste parameters. This causes the new product’s price to be lower and thus reduces the firm’s incentive to improve product quality. Consequently, the monopoly seller chooses a lower product quality than the monopoly renter chooses when the ratio of the replacement purchases is sufficiently low. This suggests that allowing the monopolist to rent its product to consumers may accelerate the quality improvement.

5. Conclusion

This paper has constructed a model analyzing the durable goods monopolist’s quality choice in the sales case and the rental case. In the case where the unit cost is sufficiently elastic with respect to quality, the monopoly seller chooses a higher product quality than the monopoly renter chooses. In the case where the unit cost is not sufficiently elastic with respect to quality, the monopoly seller may choose a lower product than the monopoly renter chooses. This suggests that, in this case, prohibiting the rental policy makes the monopolist choose a lower product. The existence of transaction costs in the secondhand market reduces the monopoly
seller's incentive to improve the product quality since it relaxes the intensity of competition between the monopolist and the secondhand market. Therefore, allowing the monopolist to rent its product to consumers may accelerate the quality improvement.

Appendix

In this appendix, we examine the sufficient conditions for profit maximization. The sufficient conditions for the maximization of the monopoly seller's second-period profits are

\[
\frac{\partial^2 \pi^s}{\partial q_2^s} = -2k^s < 0, \quad (A1)
\]

\[
\frac{\partial^2 \pi^s}{\partial k \partial q_2} = \frac{k^s}{k^s}[c'(k^s) - \frac{c(k^s)}{k^s}]F'(k^s) > 0. \quad (A2)
\]

Since assumption (8) establishes \(k^s > 1\), (A1) clearly holds. Also, from (8) and (9),

\[
q_2^s = \left[\frac{c'(k^s)}{k^s} - \frac{c(k^s)}{k^s}\right] - \frac{q_1^s}{k^s} > 0,
\]

thus, we obtain

\[
c'(k^s) - \frac{c(k^s)}{k^s} > 0.
\]

Therefore, assumption (7) insures that condition (A2) holds.

Similarly, we can show that the sufficient conditions for the maximization of the monopoly renter's second-period profits are satisfied.

1) Renting may create some serious hazards. If the consumers' consumption mode (maintenance, care, etc.) matters, the monopolist must monitor at the end of each period the exact condition of the good. Such a monitoring technology, however, may be extremely costly. There are also numerous papers concerning whether a durable goods monopolist, in reality, prefers renting to selling (see, e.g., Bucovestsky and Chilton (1986), Malug, Solow and Kahn (1988), Bhatt (1989), DeGrauba (1994) and Waldman (1997)).

2) This paper's model is an extension of Mann's (1992) and Kinokuni's (2000). Mann considers that the monopoly seller's choice variable is only a quantity. Kinokuni's model allows the monopoly seller to choose both levels of quantity and quality. This paper compares the quality level the monopoly seller chooses with that the monopoly renter chooses.

3) In the case \(s_1 = s_2\), this model coincides with Bulow's (1982). Therefore, attention in this paper is restricted to the case \(s_1 < s_2\).

4) This paper's model is same as Kinokuni's (2000) when \(\theta = 0\) and \(\bar{\theta} = 1\).

5) We do not consider the case where consumers of type \(\theta = \theta_1\) do no purchase the unit of quality \(s_1\) in period 1 and purchase the unit of quality \(s_2\) in period 2. To put it more precisely, it is assumed that

\[
\theta s_1 + \delta P_{1\theta} - P_1 \geq \delta (s_2 - P_2) \text{ for all } \theta \geq \theta_1.
\]

This is, we assume the rationing rule where consumers who have a higher taste parameter purchase the output in period 1.

6) In Waldman's (1996) model, it is assumed that consumers 1 buy a new good in period 2 and consumers 2 buy the old good. In this paper, as is often the case, consumers 2 may also purchase the new good in period 2. This is the case in Figure 1, i.e., some consumers who buy nothing in period...
In the case $\theta_1 < \theta_{eq}$, the quantity supplied of the old unit is $\bar{\theta} - \theta_{eq}$ and the quantity demanded is $\bar{\theta}_1 - \theta_0$. Consequently, the clearing price in the secondhand market is the same as (5). Note that $\theta_{on} = \theta_{eq}$.

For example, the cost function of the form $c(k) = \gamma k^a$ where $\gamma > 0$ and $a > 1$ satisfies assumptions (2), (7) and (8).

Motta (1993), using a duopoly model, shows that firms who face more severe competition choose more differentiated products.

References