Excessive or Insufficient Entry under Cournot Oligopoly with Product Differentiation*

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Abstract

We analyze Cournot oligopoly model with product differentiation, in which demand and cost functions are specified. We establish the following results. In the case of highly differentiated brands, an increase in the degree of product differentiation enhances product diversity effect and reduces business-stealing effect. If entry cost is large, then the former effect exceeds the latter one, therefore, entry is insufficient. In the remaining case, since it facilities both product diversity and business-stealing effects, the latter exceeds the former, therefore, entry is excessive.

Keywords: Cournot Oligopoly with product differentiation, excessive or insufficient entry, product diversity effect, business-stealing effect

JEL Classification numbers: D43; L13

1. Introduction

Under Cournot oligopolistic competition in a homogenous product market, Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) state that a marginal increase in the number of firms at the free entry equilibrium causes the loss of social welfare if the strategic interaction among firms is strategic substitute. This is called as excess entry theorem. Besley and Suzumura (1992), Suzumura (1992), and Okuno-Fujiwara and Suzumura (1993) show the validity of this theorem by analyzing dynamic framework with R&D activity. Ohkawa and Okamura (1999) shows that this theorem is globally valid, and that if strategic interaction among firms is strategic substitute (complement), then firms enter into the market excessively (insufficiently).

Following Mankiw and Whinston (1986), we show the basic concept about excess entry theorem. An entrant causes the decrease in each incumbent' output level that leads to reduce the level of social welfare (business stealing effect) because strategic substitutability holds,

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while it makes profits that improve the welfare. Because zero profit condition is satisfied at the free-entry equilibrium, the latter effect disappears at this equilibrium, so that entry harms the social welfare.

In a differentiated product market, several articles, for instance Dixit and Stiglitz (1977), Mankiw and Whinston (1986), and Anderson, de Palma, and Nesterov (1995), state that free entry can lead to social inefficiency, that is, firms enter into the differentiated market either excessively or insufficiently. The result is derived from the analyses using monopolistic competition model. The intuition of this result is as follows: According to Suzumura (1995), product differentiation adds the effect of product diversity to above two effects. This effect improves the welfare if consumers have preference for product diversity. At the free-entry equilibrium, given business-stealing effect caused by strategic substitutability, then two contrary effects are invoked: business-stealing effect that harms welfare and product diversity effect that improves it. Therefore, entry (increase in the number of variety) may be socially excessive or insufficient.

Certainly the model of monopolistic competition is convenient for the analysis of product differentiation, but it does not deal with strategic interaction among firms explicitly. In order to compare with the result derived from homogenous goods model directly, we have to analyze the Cournot oligopoly model with product differentiation. This direct comparison enables us to understand the working of the above two contrary effects through product differentiation.

In this paper, we analyze excessive or insufficient entry problem by using Cournot oligopoly model with product differentiation. We specify the demand and cost functions in order to make the analysis easier. We show that if entry cost is large and if each variety is highly differentiated, then firms enter into the market insufficiently, and that otherwise, then firms enter into the market excessively. We also show that a change in the degree of product differentiation affects not only product diversity effect but also business-stealing effect, so that entry is socially inefficient.

The rest of this paper is organized as follows: In section 2, we present the specified Cournot oligopoly model with product differentiation and establish that entry is excessive or insufficient. Section 3 presents the explanation of excess or insufficient entry in the case of product differentiation. Conclusions are given in section 4.

2. The Analysis

Suppose that there are n brands in a differential market. Following Sakai (1990), we specify the utility function for a representative consumer as follows:

$$U(q_1, \cdots, q_n)=\alpha \sum_{i-1}^{n} q_i-\frac{1}{2}\left(\sum_{i=1}^{n} q_i^2+\theta \sum_{i=1}^{n} \sum_{i \neq j} q_i q_j\right), \tag{1}$$

where q_i is a quantity of ith brand's demand, θ is measure of product differentiation, and

$$0<\theta \leqq 1. \tag{2}$$

The larger the level of θ is, the less differentiated the goods is. The consumer's behavior that is maximizes consumer surplus,

$$CS = U - \sum_{i=1}^{n} p_i q_i \tag{3}$$

yields the following inverse demand function:

$$p_i = \alpha q_i - \theta \sum_{i \neq j} q_j.$$

Each identical firm faces to the following optimal problem

$$\max_{q_i} . \pi_i = p_i q_i - c q_i - f,$$

where c is common unit cost and f is common fixed entry cost. Adding up the first order condition of each firm yields

$$n(\alpha - c) - 2\sum_{i=1}^{n} q_i - \theta(n-1)\sum_{i=1}^{n} q_i = 0. \tag{4}$$

Assuming symmetric Cournot equilibrium, i. e., $q_i = q^*$, from (4), we obtain

$$q^* = \frac{\alpha - c}{x+1}, \tag{5}$$

where $x = 1 + (n-1)\theta \geqq 1$. Equilibrium price and profit of each firm are respectively

$$p^* = \frac{\alpha + xc}{x+1},$$
$$\pi^* = \left(\frac{\alpha - c}{x+1}\right) - f. \tag{6}$$

From (6), the free-entry Cournot equilibrium satisfies the following zero profit condition:

$$\left(\frac{\alpha - c}{x+1}\right)^2 = f.$$

Therefore, the free-entry Cournot equilibrium output level of each firm, q^{FE} and the free-entry Cournot equilibrium number of brands (firms), n^{FE} are as follows:

$$q^{FE} = \sqrt{f}$$
$$n^{FE} = \frac{1}{\theta}\left(\frac{\alpha - c}{\sqrt{f}} + \theta - 2\right) \tag{7}$$

At the equilibrium where the number of brands is fixed, from (3), (5), and (6), consumer surplus, CS and producer surplus, PS can be respectively expressed as

$$CS = \frac{1}{2}\sum_{i=1}^{n} q_i^2 + \frac{1}{2}\theta \sum_{i=1}^{n}\sum_{j \neq i} q_i q_j = \frac{1}{2} nx \left(\frac{\alpha - c}{x+1}\right), \text{ and} \tag{8}$$

$$PS = n\left(\frac{\alpha - c}{x+1}\right) - nf. \tag{9}$$

Social planner faces to the welfare-maximizing problem controlling the number of brands. We can define social welfare, W as the sum of consumer and producer surpluses. From (8) and (9), the first and second order conditions of the above problem are

$$\frac{\partial W}{\partial n} = \frac{\partial CS}{\partial n} + \frac{\partial PS}{\partial n} = \frac{(\alpha - c)^2}{2(x+1)^3}[\theta(1-\theta)n + (\theta-2)(\theta-3)] - f = 0, \text{ and} \tag{10}$$

$$\frac{\partial^2 W}{\partial n^2}=-\frac{(\alpha-c)^2\theta}{(x+1)^4}[2\theta(1-\theta)n+(2-\theta)]<0. \tag{11}$$

From (11), the following lemma is obtained.

Lemma 1: Social welfare is strictly concave function with respect to the number of brands.

Lemma 1 ensures that optimal number of brands (firms) is uniquely determined if it exists.

We assume that $\alpha-c>\sqrt{f}$. This assumption shows that the level of each firm's fixed cost is relatively smaller than market size. Let us introduce the new variable b for the relationship between $\alpha-c$ and $\sqrt{f}$ such that

$$b(\alpha-c)=\sqrt{f} . \tag{12}$$

Note that $0<b<1$. Using (12), we can transform the RHS of (10) as

$$\frac{\partial W}{\partial n}=\frac{(\alpha-c)^2}{2(x+1)^3}[\theta(1-\theta)n+(\theta-2)(\theta-3)-2(x+1)^3b^2]. \tag{13}$$

Define the terms in the parenthesis on the RHS of (13) as Z. Because $x\geqq 1$, the following equation are satisfied from (13):

$$sgn\left(\frac{\partial W}{\partial n}\right)=sgn(Z). \tag{14}$$

From lemma 1 and (14), we can obtain the following lemma.

Lemma 2: If the sign of Z is negative (positive) at the free-entry Cournot equilibrium, then firms enter into the market excessively (insufficiently).

(*proof.*) If Z is negative (positive), then the sign of the derivative of W with respect to n is negative (positive) at the free-entry Cournot equilibrium. Because W is strictly concave function with respect to n, if optimal number of firms satisfying (10) exists, then it is smaller (larger) than the free-entry equilibrium number of firms □

From lemma 2 we focus only on the sign of Z at the free-entry equilibrium. Substituting (12) into (7) yields

$$n^{FE}=\frac{1-2b+b\theta}{b\theta}. \tag{15}$$

Because $n^{FE}\geqq 1$, b must be satisfied the following condition:

$$0<b<\frac{1}{2}. \tag{16}$$

Substituting (15) into Z, we obtain

$$Z=\frac{2b(2-\theta)-(1+\theta)}{b}. \tag{17}$$

From (16), the sign of Z depends on that of the numerator on the RHS of (17). Therefore, if $b\geq(<)\frac{1+\theta}{2(2-\theta)}=f(\theta)$, then $Z\geq(<)0$. Considering (2) and (16), we can draw the graph of b

$=f(\theta)$ as follows:

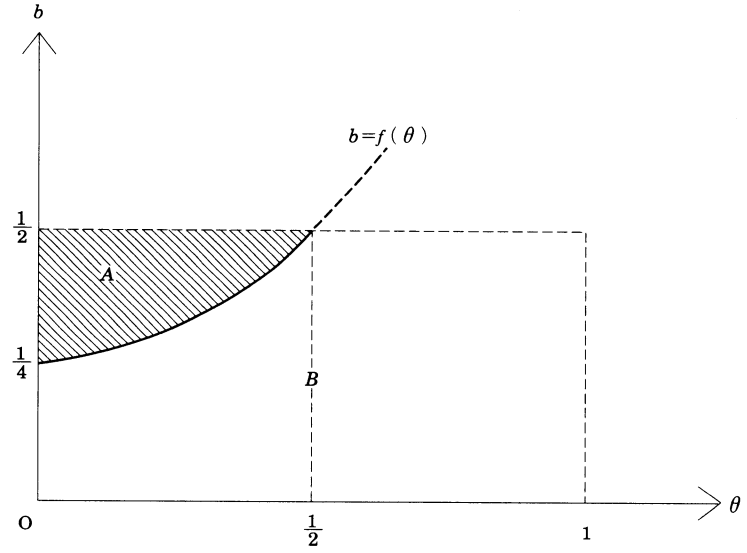

Figure 1 The graph of $b=f(\theta)$

Thus, from lemma 2, we establish the following result:

Proposition: If the pair (b, θ) is in the domain A (B) in the figure 1, then each firm enters into the market insufficiently (excessively).

The parameter b can be interpreted as the level of fixed entry cost given market size and unit cost. So the proposition means that if entry cost is not very large, then entry is always excessive from the social welfare viewpoint, and that if entry cost is large and if each variety is highly differentiated, then it is insufficient.

3. The Interpretation

From now, we consider the implication of the above proposition. First of all, let us consider the effect of consumer surplus on the increase in the number of brands. From (2) and (8), we obtain

$$\frac{\partial CS}{\partial n}=\frac{(\alpha-c)^2}{(x+1)^3}[n\theta(3-\theta)+(2-\theta)(1-\theta)]>0, \tag{18}$$

which means that the increase in the number of product variety always improve the welfare. This can be regarded as the effect of product diversity. Differentiating (18) with respect to θ yields

$$\frac{\partial^2 CS}{\partial n \partial \theta}=\frac{(\alpha-c)^2(n-1)\theta}{(x+1)^4}[(n-1)(\theta-6)-4]<0.$$

Above equation indicates that the higher each variety is differentiated, the more the effect of product diversity facilitates the increase in the level of welfare.

Thus, we establish the following result:

Corollary 1: Product diversity effect increases the level of consumer surplus. And the facility of product differentiation enhances this effect.

Next consider the effect of producer surplus on the increase in the number of product variety. From (9), we get

$$\frac{\partial PS}{\partial n}=\frac{(\alpha-c)^2}{(x+1)^3}(x+1-n\theta)-f. \tag{19}$$

Since the sign of above equation is generally ambiguous, it is evaluated at the free-entry equilibrium. From (2), (12), and (15), above equation can be transformed into

$$\left.\frac{\partial PS}{\partial n}\right|_{n=n^{FE}}=-\frac{2(\alpha-c)^2 n\theta}{(x+1)^3}<0.$$

This means that product diversity decreases the level of producer surplus, which can be regarded as business stealing effect. Differentiating (19) with respect to θ and evaluating at the free-entry equilibrium yield

$$\frac{\partial^2 PS}{\partial n \partial \theta}_{n=n^{FE}}=-\frac{2(\alpha-c)^2(n-1)}{(x+1)^4}\left(6-3\theta-\frac{2}{b}\right). \tag{20}$$

From (20), we obtain the following results. If $b(\geqq)<g(\theta)$, then the LHS of (20) is nonpositive (positive), where $g(\theta)=\frac{2}{3(2-\theta)}$. The graph of $b=g(\theta)$ is drawn in figure 2.

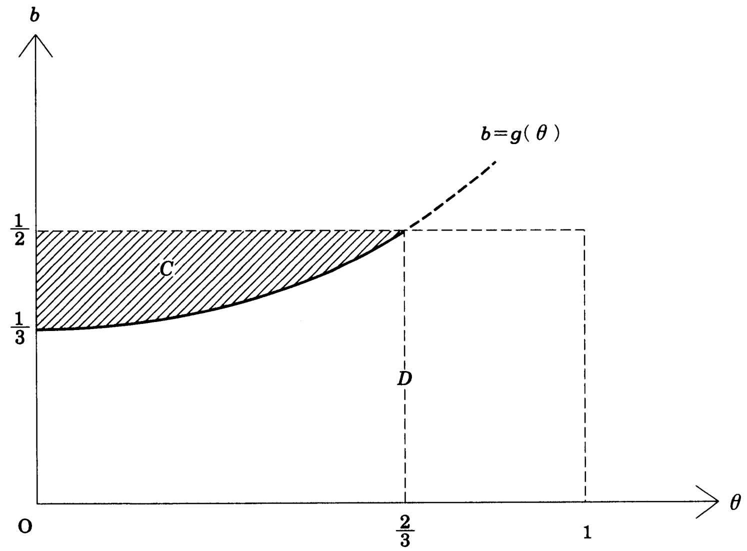

Figure 2 The graph of $b=g(\theta)$

Therefore, we obtain the following result about business-stealing effect and the relationship between this effect and the degree of product differentiation.

Corollary 2: (1) At the free-entry equilibrium, business-stealing effect always holds. (2) Suppose that the degree of product differentiation increases. If the pair (b, θ) is in the domain C (D) in figure 2, then the business-stealing effect relaxes (tightens).

The above two corollaries indicate the following intuition. Suppose that entry cost is large, that is, a few firms enter into the differentiated market, and that each brand is highly differentiated. Then, the increase in the degree of product differentiation gives rise to two effects. One is to facilitate product diversity effect that brings about the improvement of social welfare. The other is to reduce the business-stealing effect that gives rise to the decrease in the level of welfare. At the free-entry equilibrium, therefore, the product diversity effect exceeds the business-stealing effect. Thus, the increase in the number of product variety improves welfare, so that entry is insufficient from the social welfare viewpoint.

Otherwise, then the increase in the degree of product differentiation certainly enhances the product diversity effect, but may increase the business-stealing effect. At the free-entry equilibrium, therefore, the business-stealing effect exceeds the product diversity effect. Thus, the increase in the number of variety harms welfare, so that entry is excessive from the social welfare viewpoint.

The important point of this intuition is a role of product differentiation. Previous researches emphasize the importance of product diversity effect in a differentiated market and overlook the change of the scale of the business-stealing effect through an increase in the degree of product differentiation. In this paper, we show that the scale of business-stealing effect as well as that of product diversity effect depends on the degree of product differentiation.

4. Concluding Remarks

We analyze Cournot oligopoly model with product differentiation, in which demand and cost functions are linear. We show that if entry cost is large, and if each brand is highly differentiated, then entry is insufficient, and that otherwise, then it is excessive. We also show the implication of excessive or insufficient entry. This is as follows: In the case of highly differentiated brands, an increase in the degree of product differentiation enhances product diversity effect and reduces business-stealing effect. If entry cost is large, then the former effect exceeds the latter one, therefore, entry is insufficient. In the remaining case, since it facilities both product diversity and business-stealing effects, the latter is apt to exceed the former. Therefore, entry may be excessive.

We will point out the directions of further research. First, Bertrand competition with product differentiation brings about either excessive entry or not. Second, second best outcome of Bertrand competition is compared with that of Cournot competition.

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